COMMUNITY-BASED MATRIX REORDERING FOR SPARSE LINEAR ALGEBRA OPTIMIZATION

Vignesh Balaji Neal Crago Aamer Jaleel Stephen W. Keckler

PROBLEM – SUBOPTIMAL SPARSE LINEAR ALGEBRA PERFORMANCE

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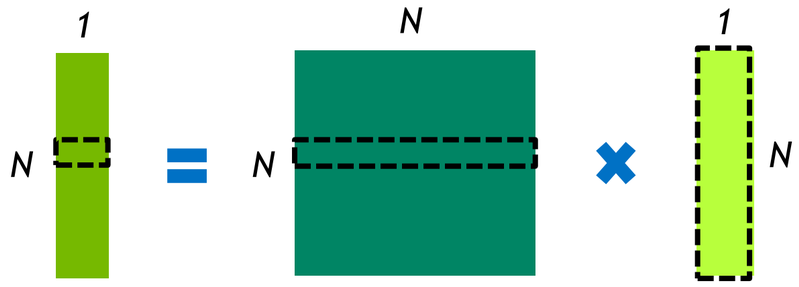
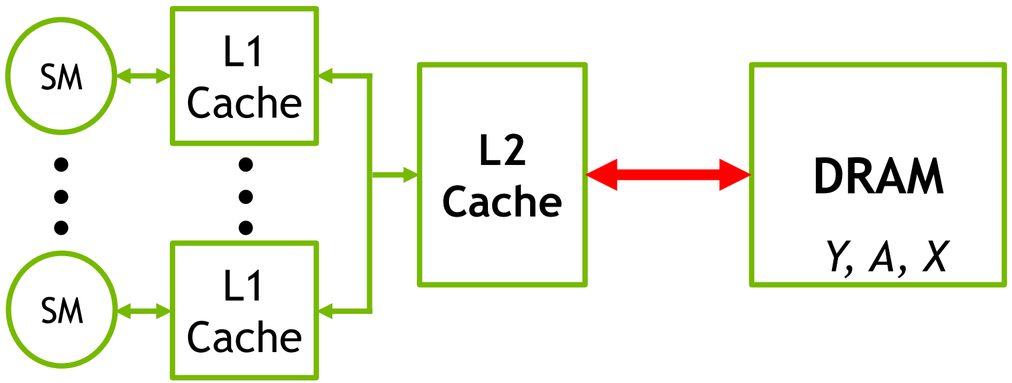


Sparse Matrix Vector Multiplication (SpMV)



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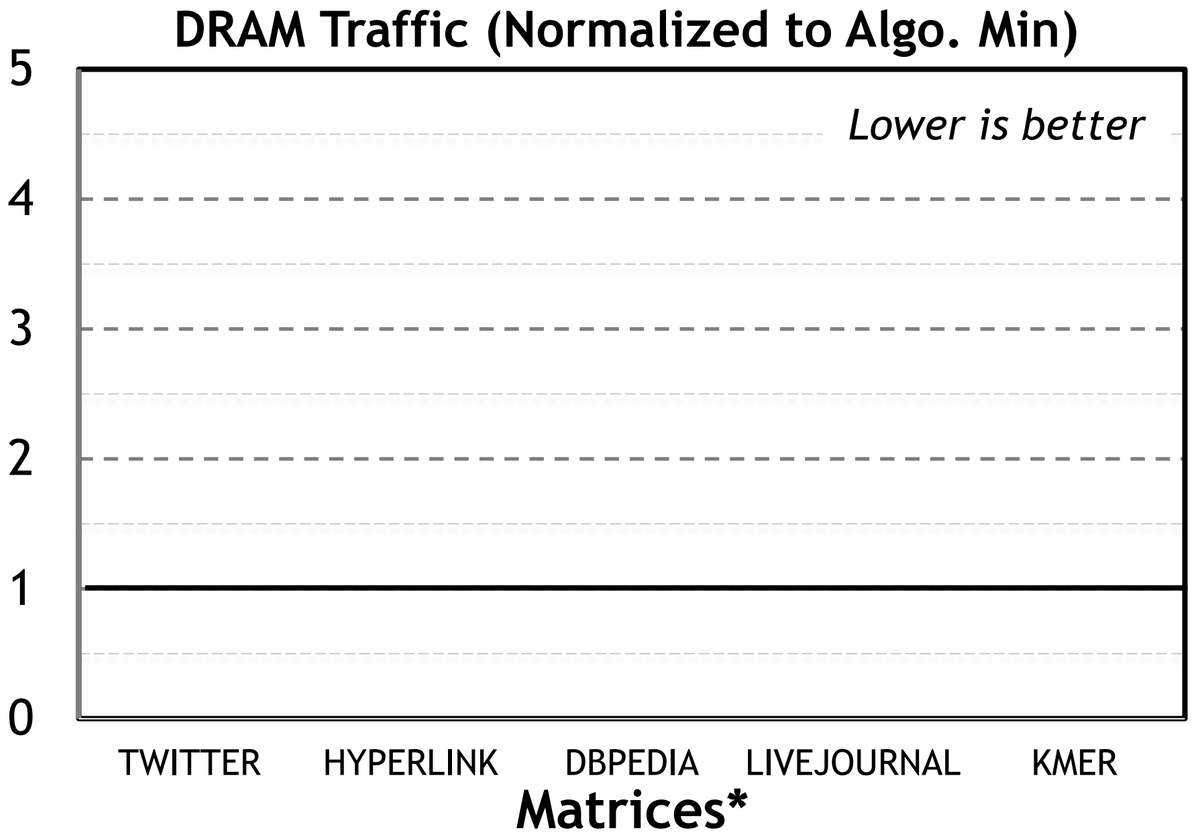
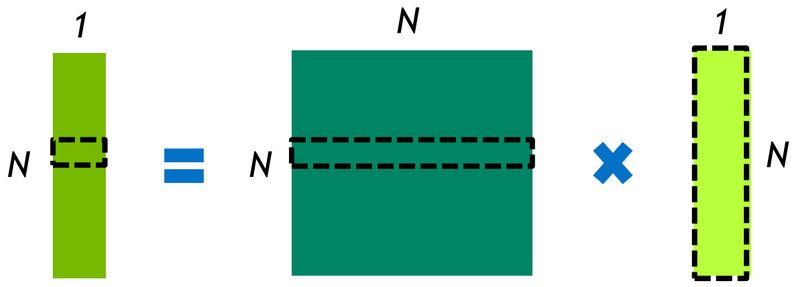
Characterization on NVIDIA A6000 GPU





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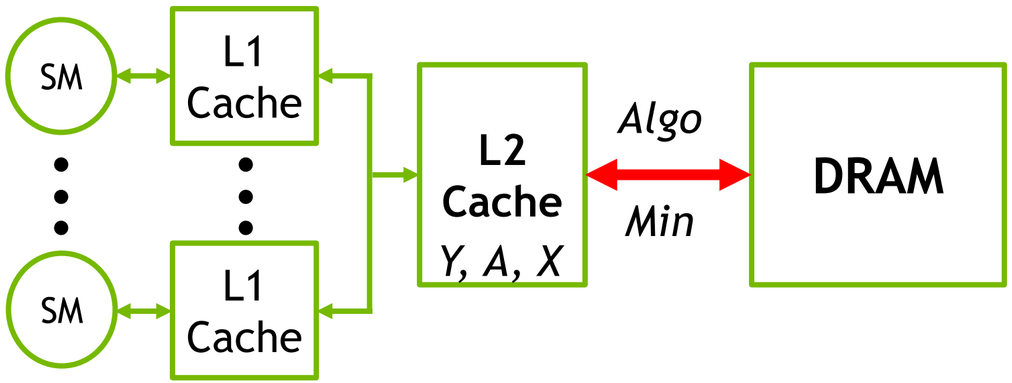

**Subset of matrices evaluated*

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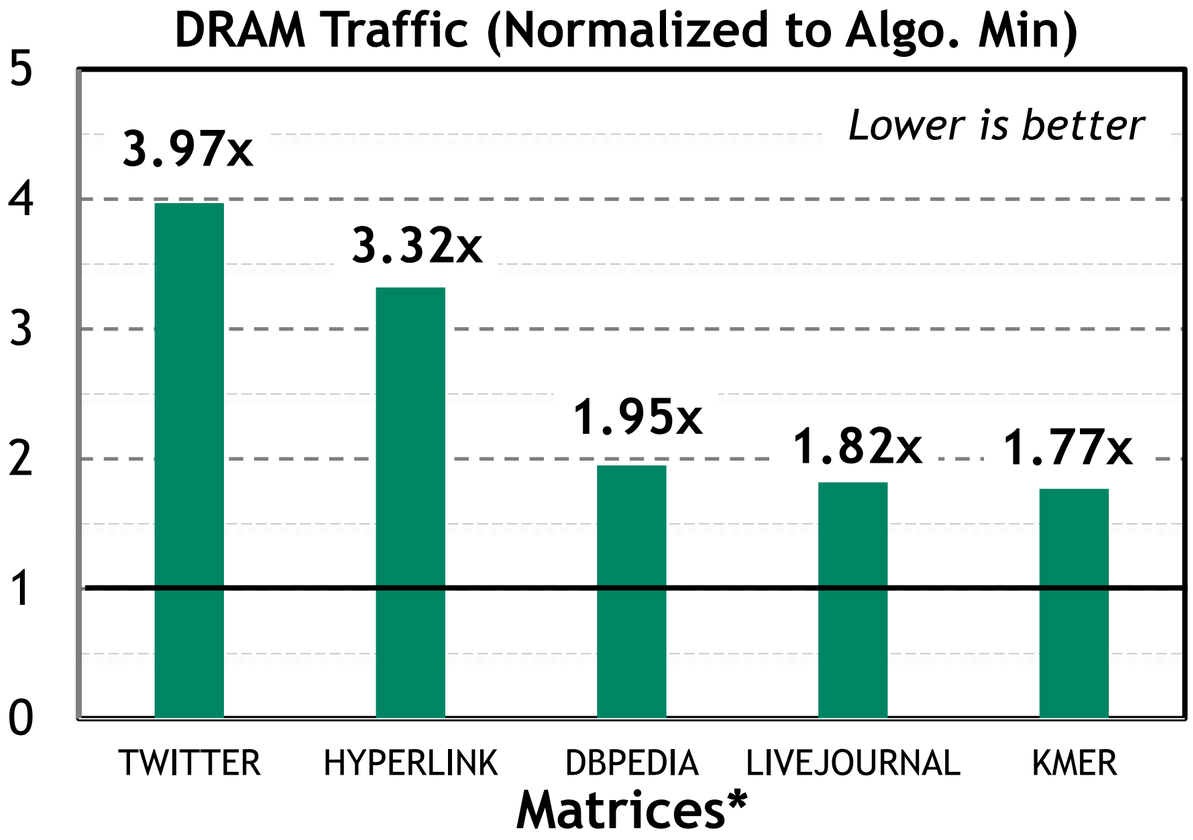

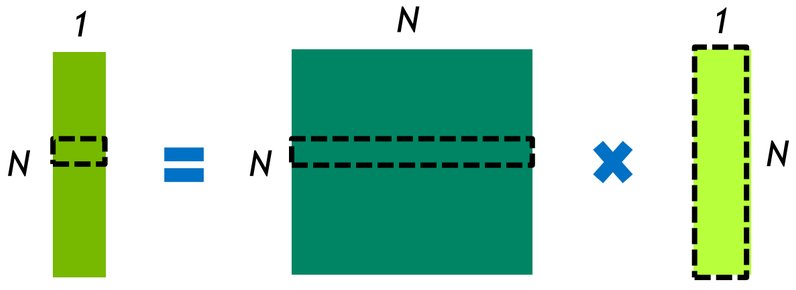

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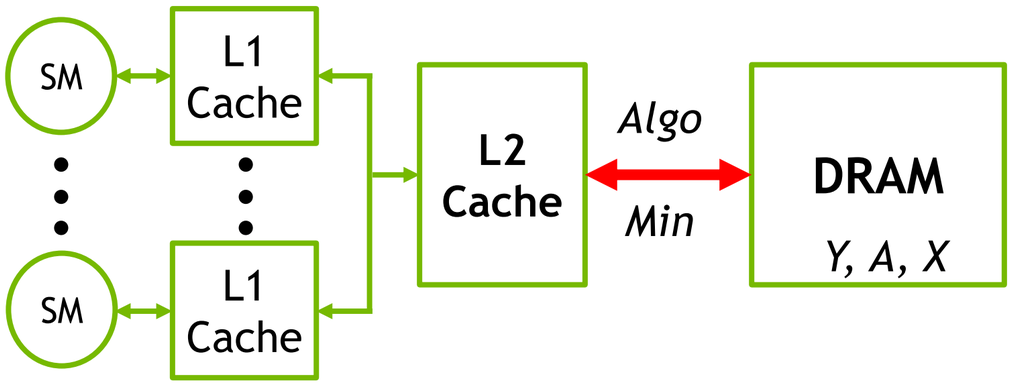

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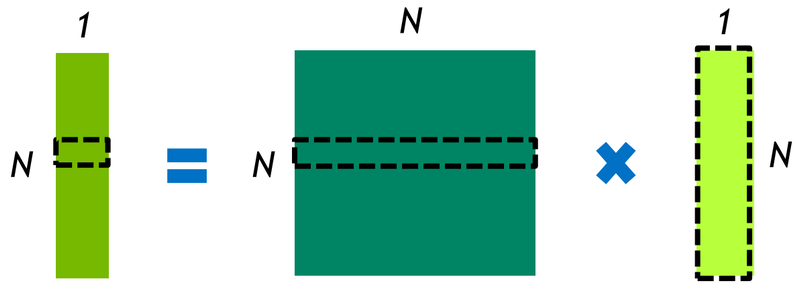

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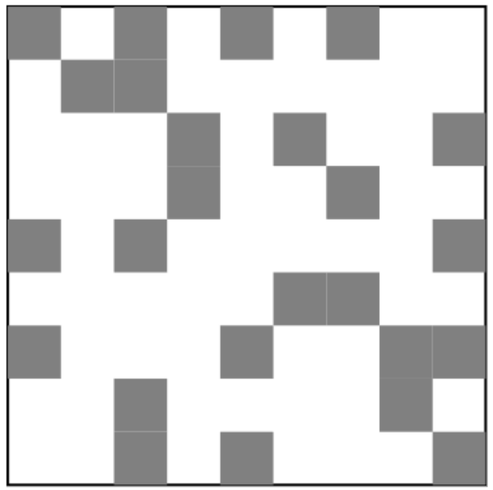
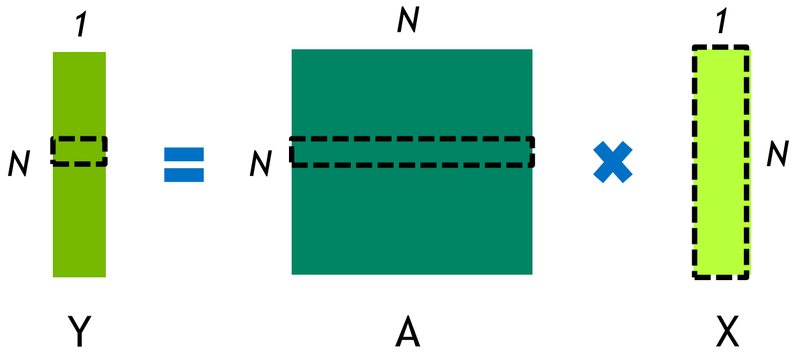
Typically, greater than 99% of entries are zeros





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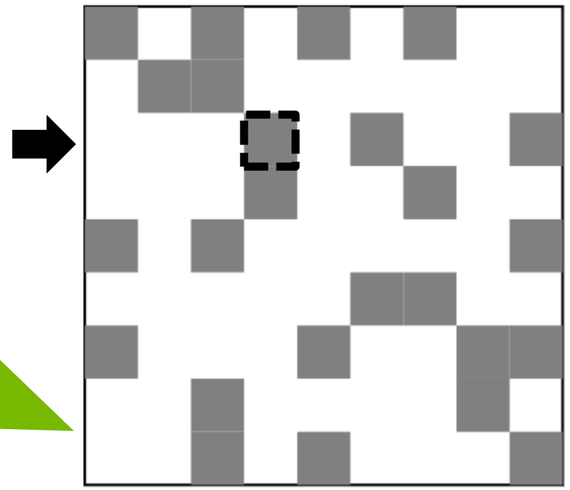

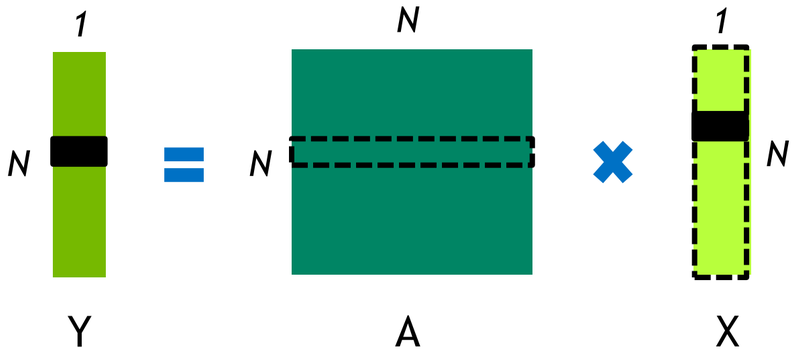

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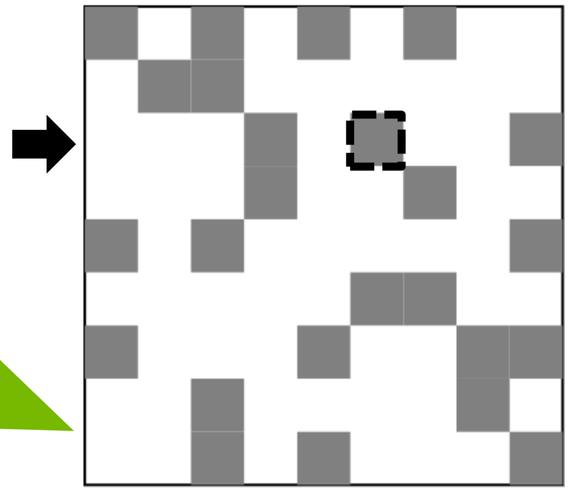

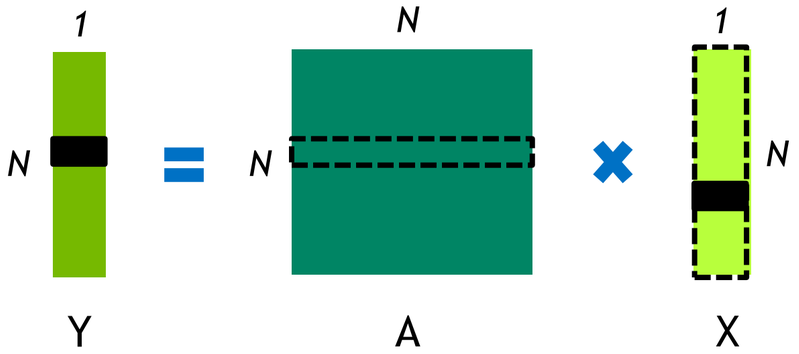

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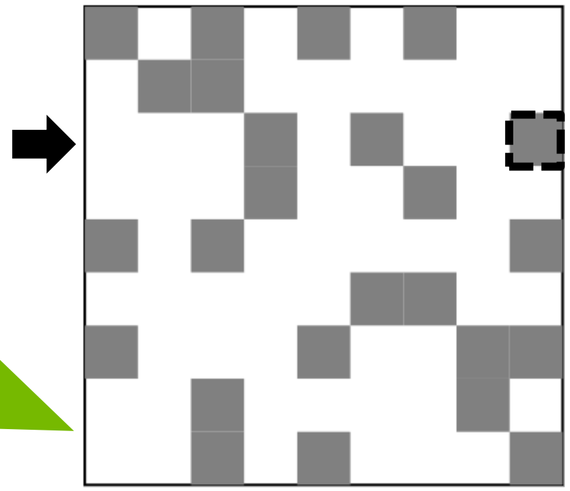
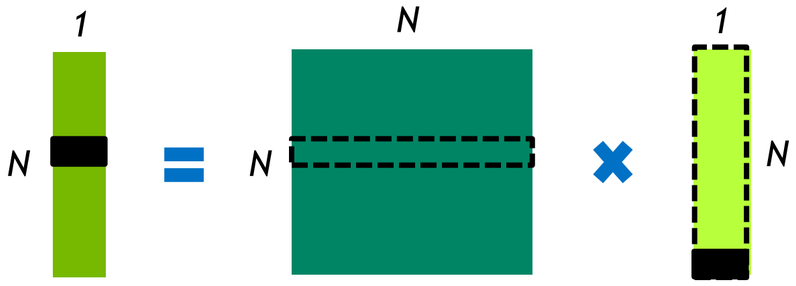
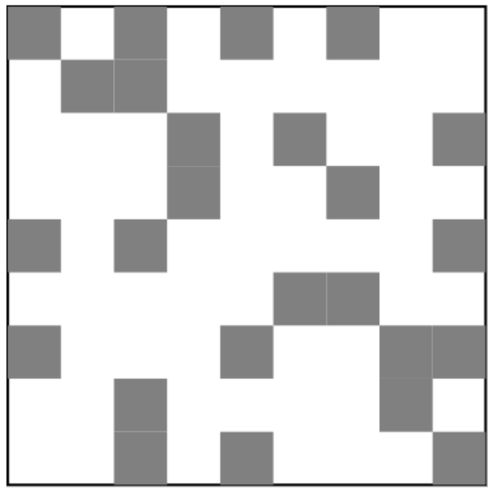




Sparse Matrix Vector Multiplication (SpMV)

Compressed Representations of sparse matrices lead to fine-grained, irregular accesses to Input Vector (X)

IMPROVING REGULARITY WITH MATRIX REORDERING



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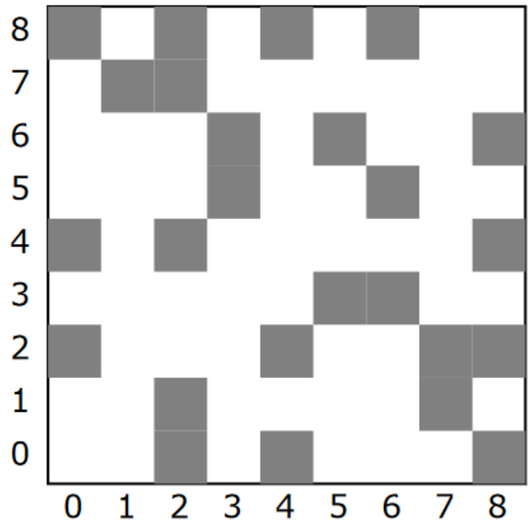

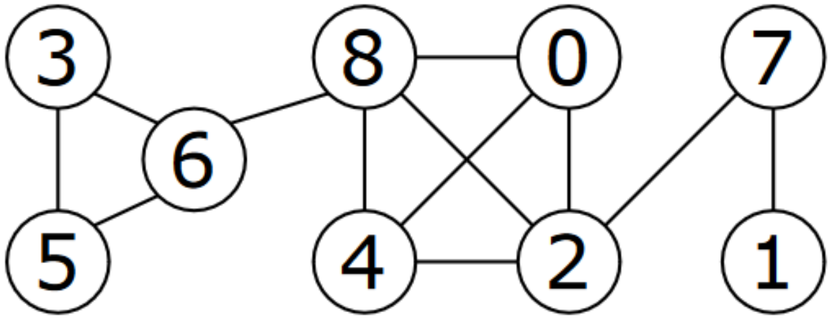

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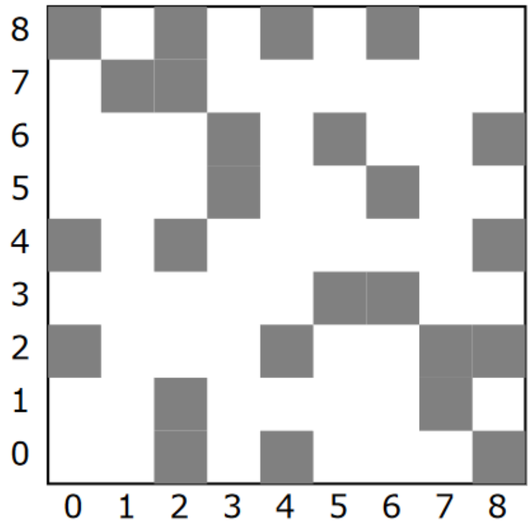

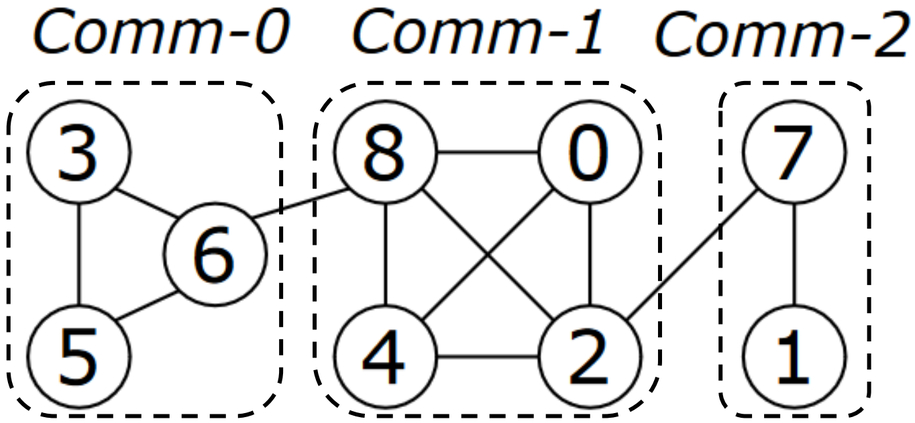

Many real-world networks exhibit community structure:

- Social Networks
- Web Crawls
- Biological Networks
- Knowledge graphs
- …

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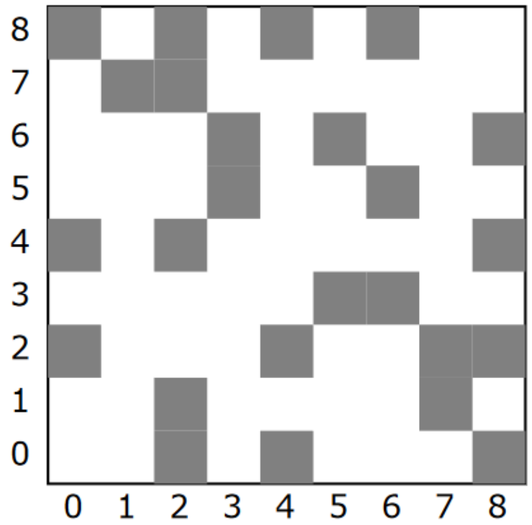

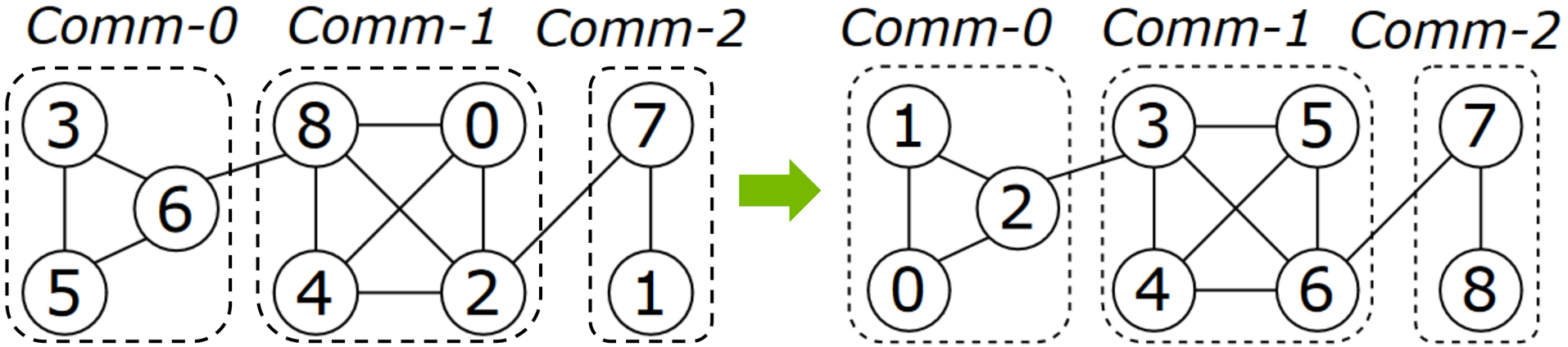

Relabel Matrix
Rows and Cols

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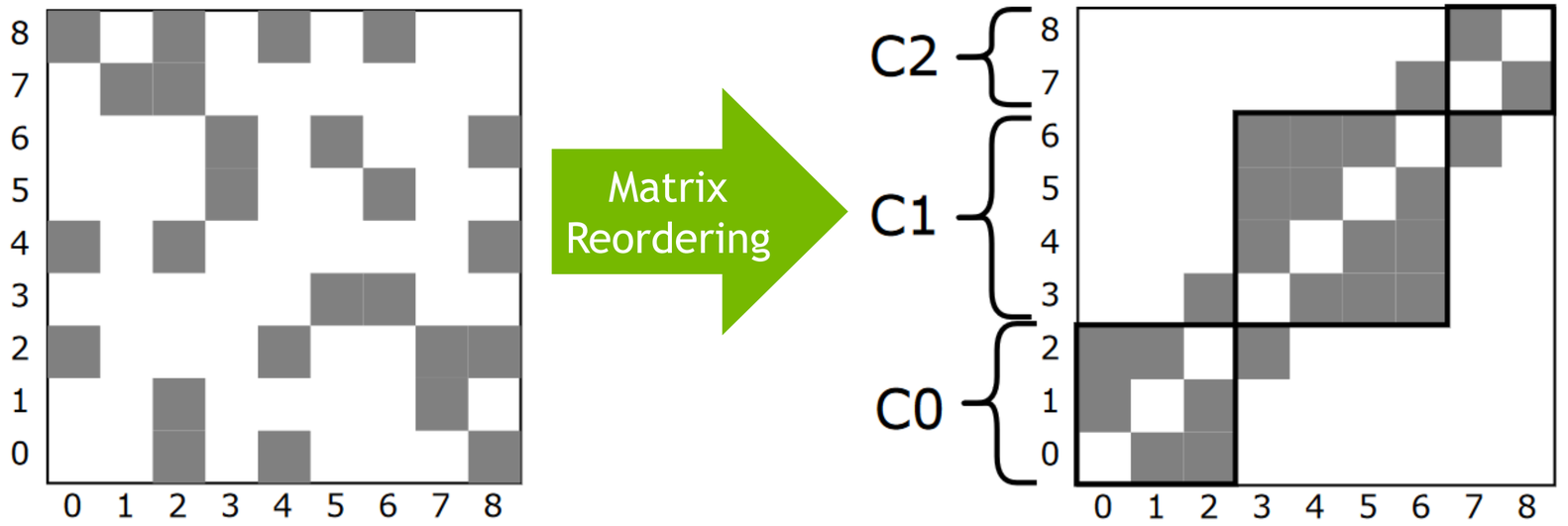

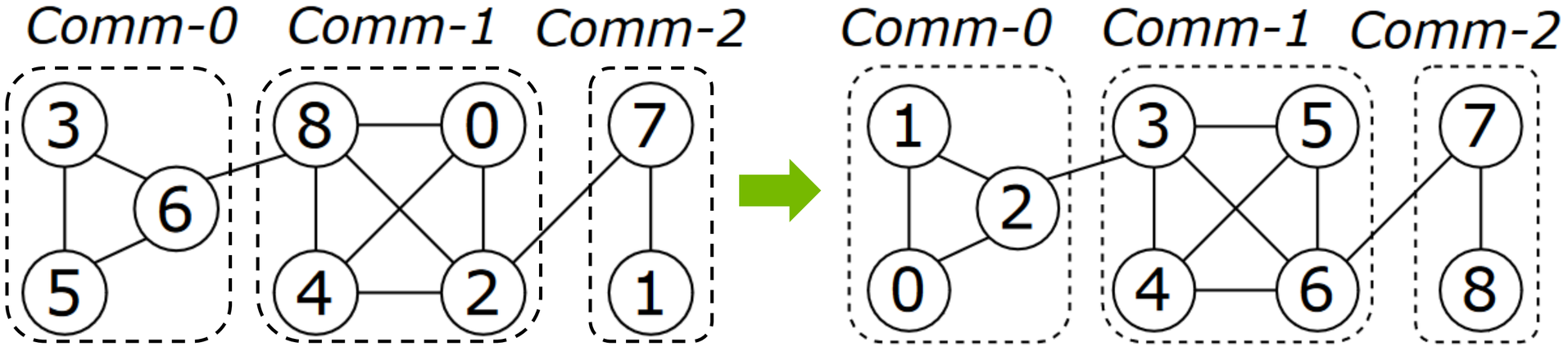

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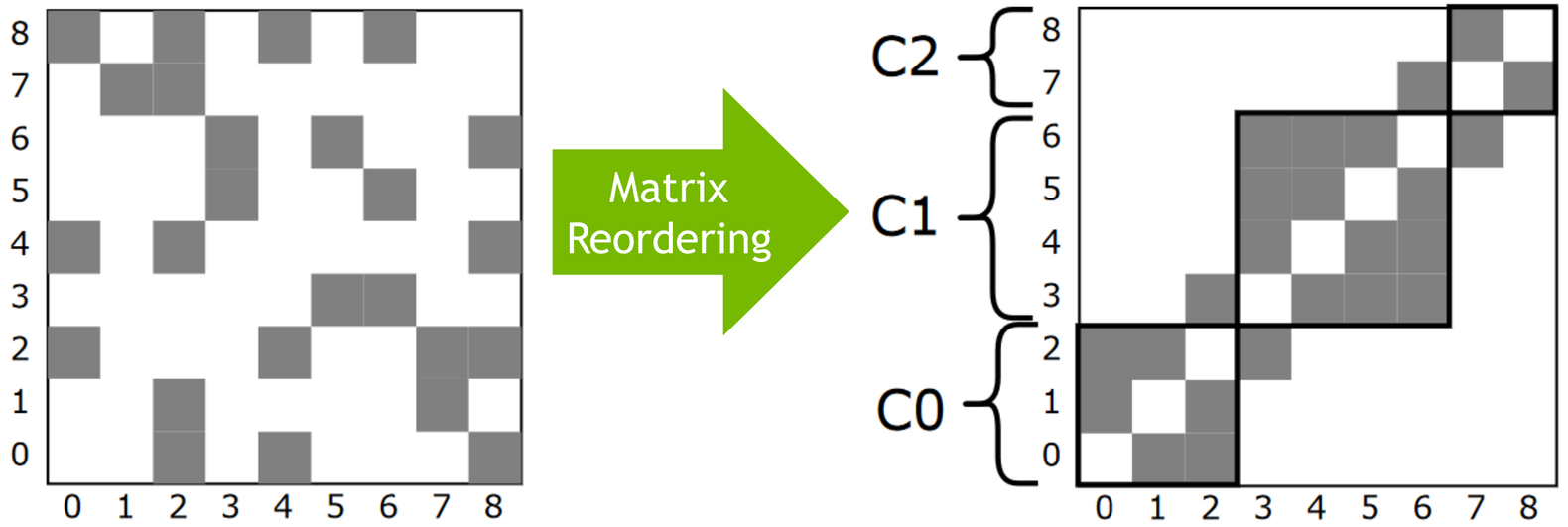

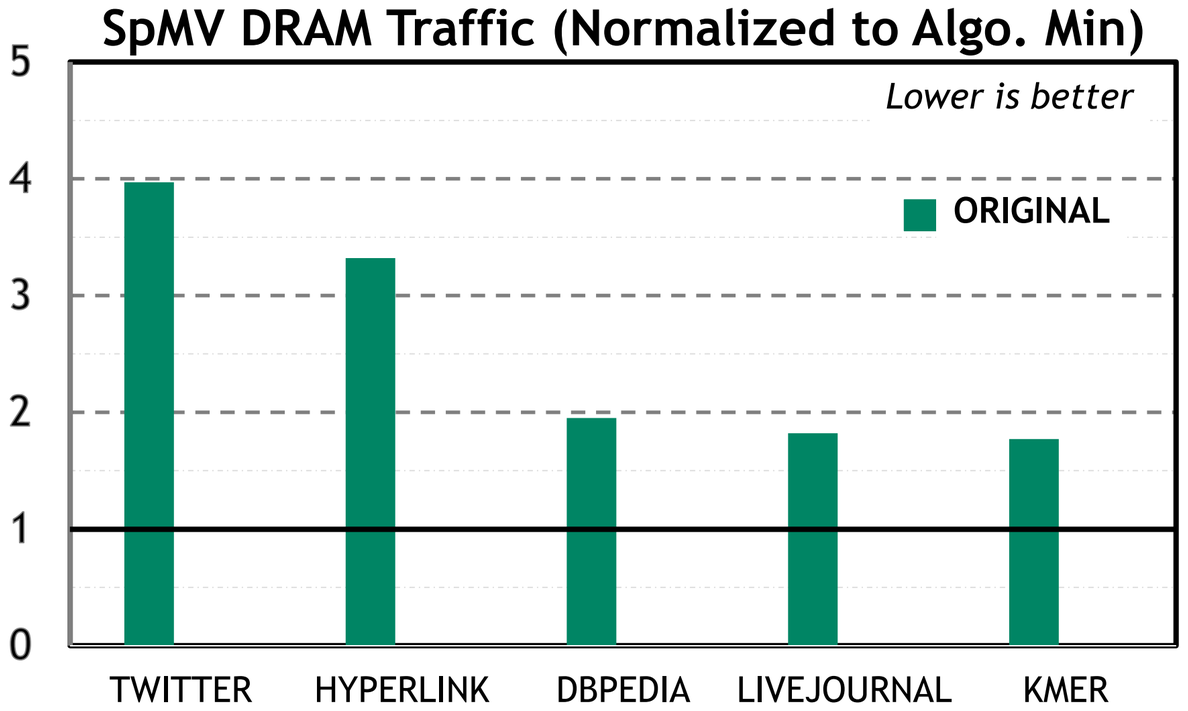

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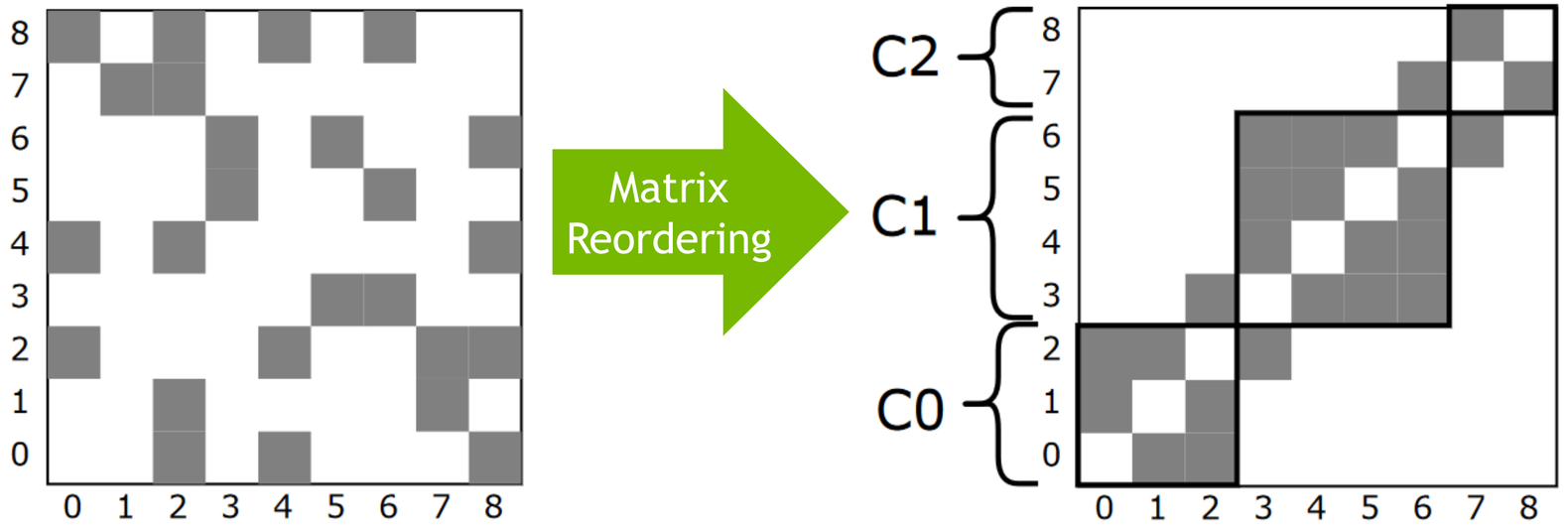

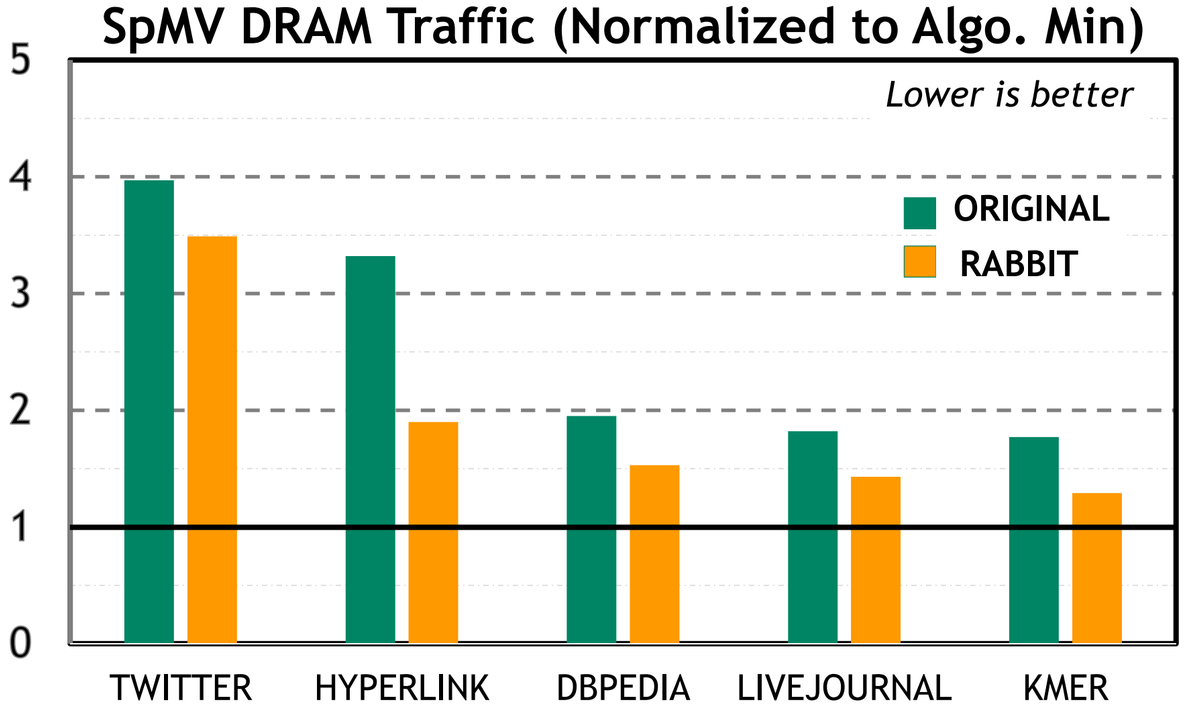

OUTLINE

- **Matrix Reordering Improves Locality** ✓
- Methodology for Evaluating Reordering Techniques
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EVALUATION METHODOLOGY

HARDWARE: NVIDIA A6000 GPU

L2 Cache	DRAM Bandwidth	Mem Capacity
6MB	768GB/s	48GB

SOFTWARE: NVIDIA cuSPARSE library (v11.8)

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The KONECT Project

Web Data Commons

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Our final input set comprises of 50 matrices spanning diverse domains

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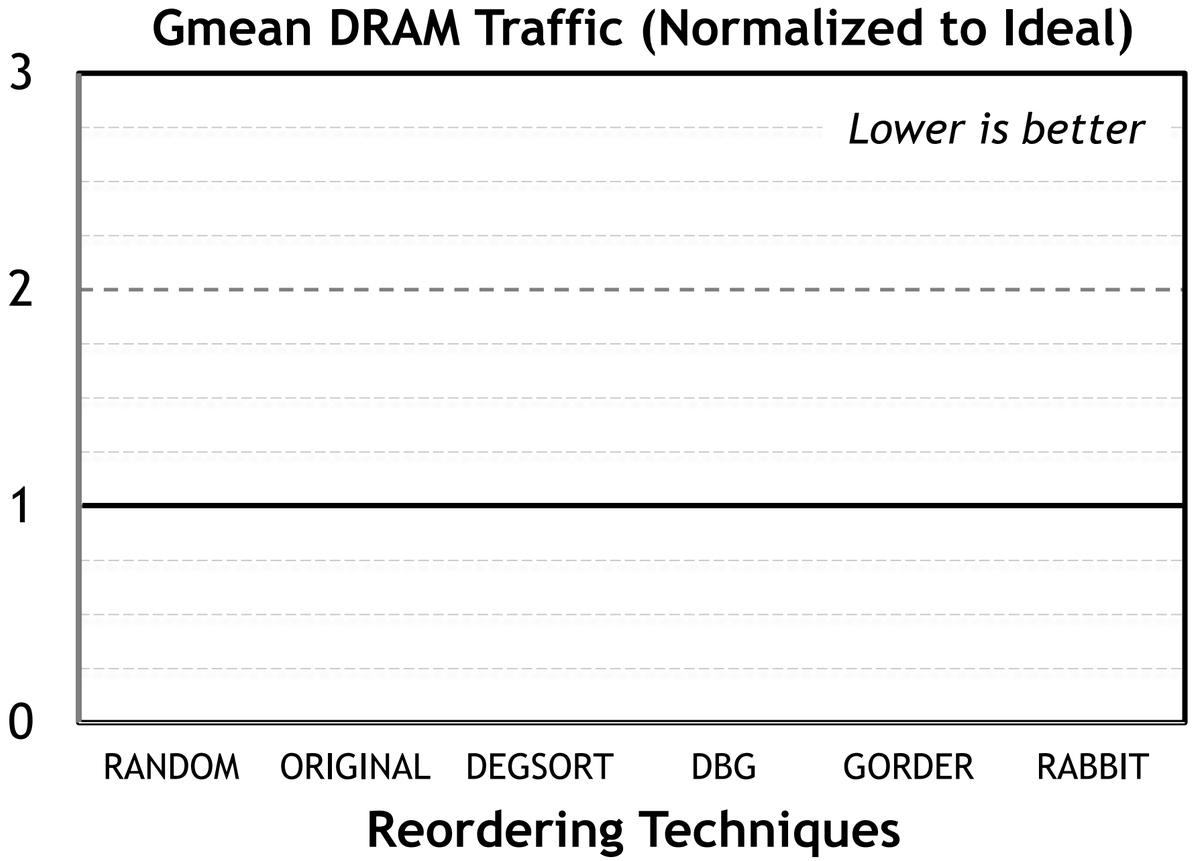
cuSPARSE SpMV on NVIDIA A6000 GPU



Ideal Traffic = Algorithmic Minimum DRAM Transfers

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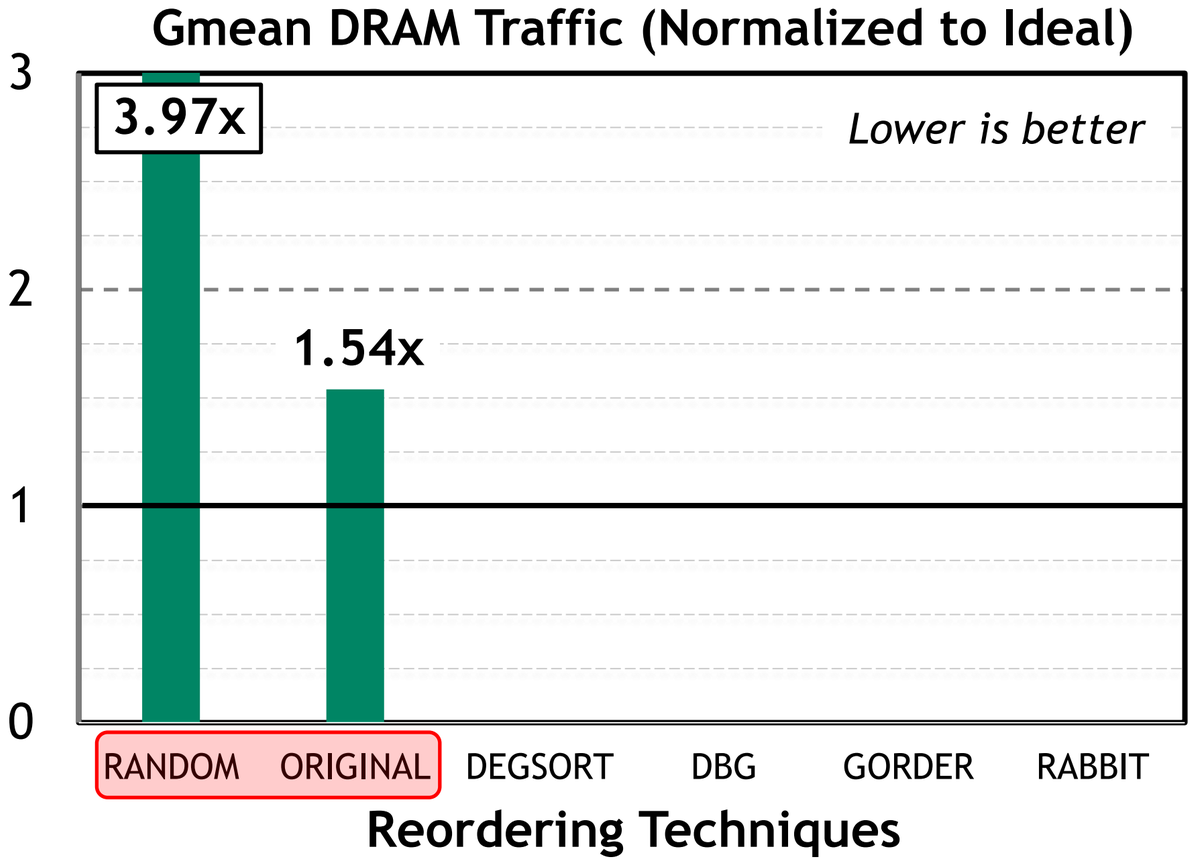

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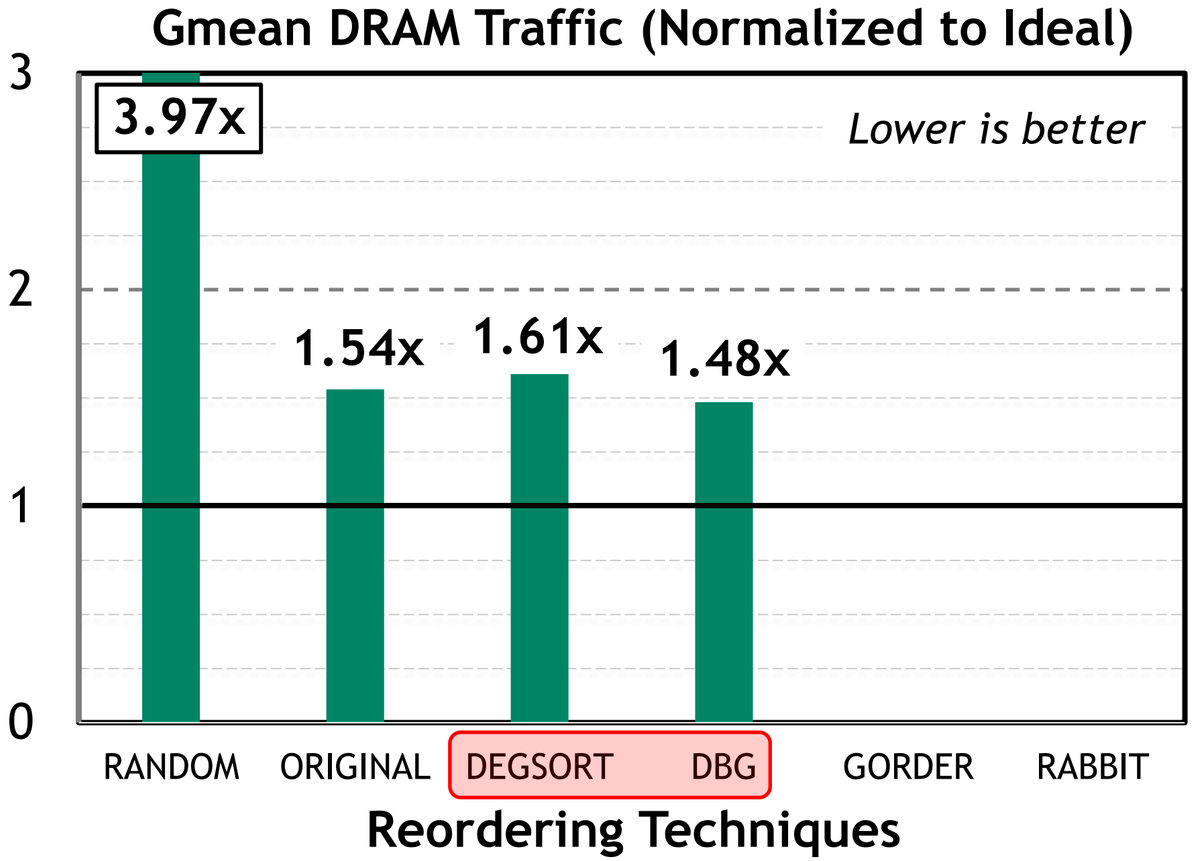

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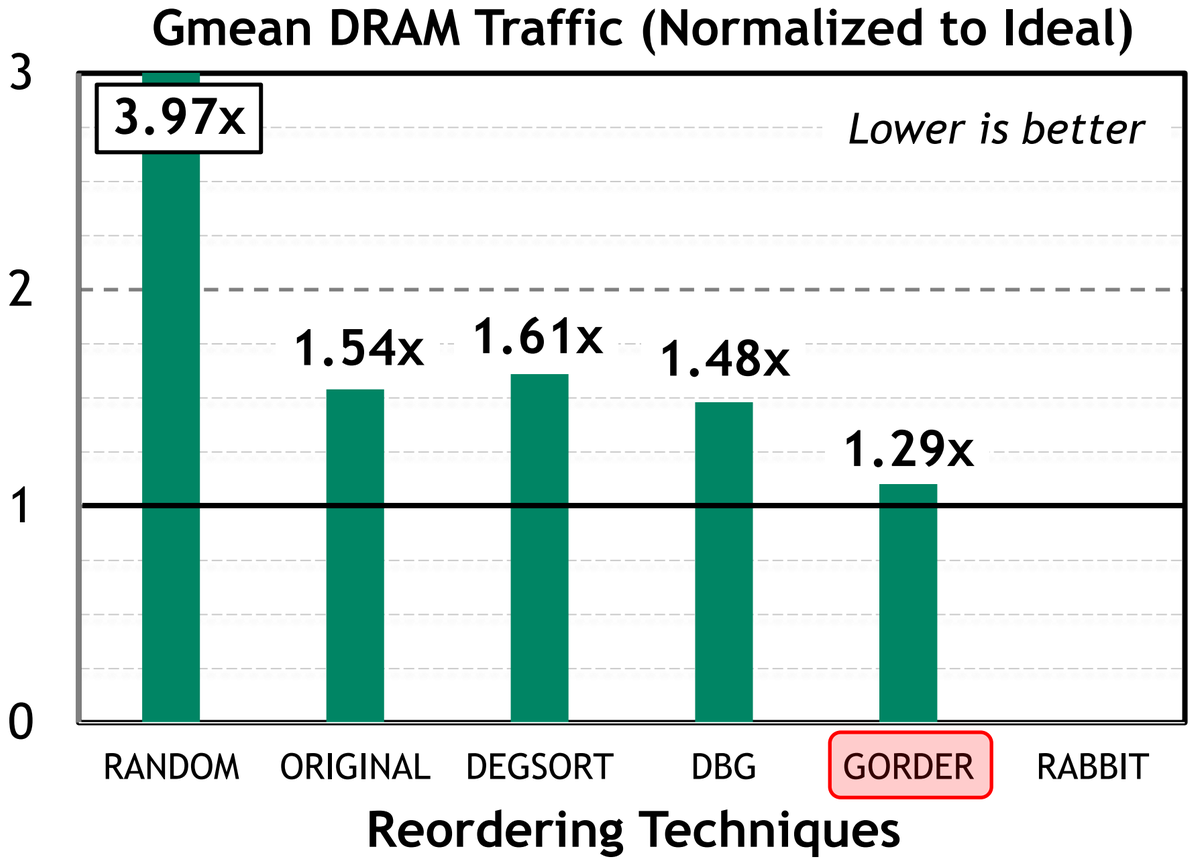

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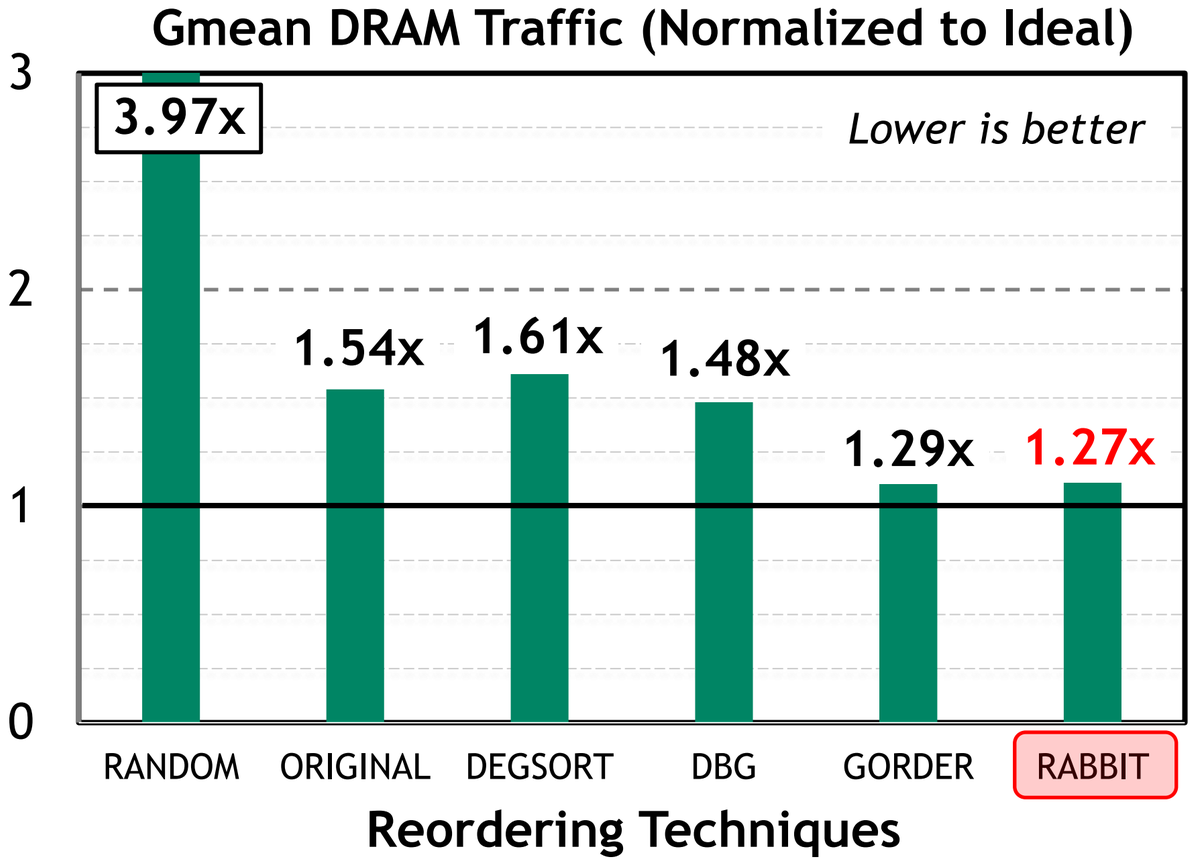

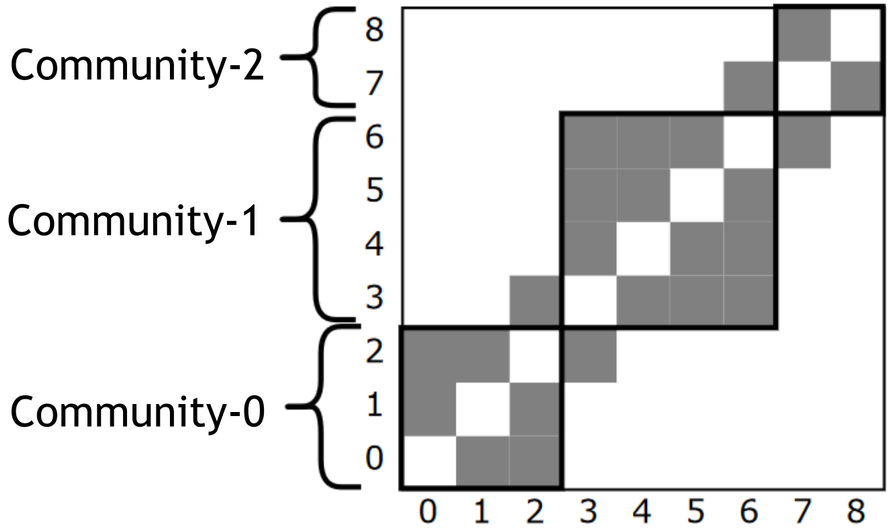

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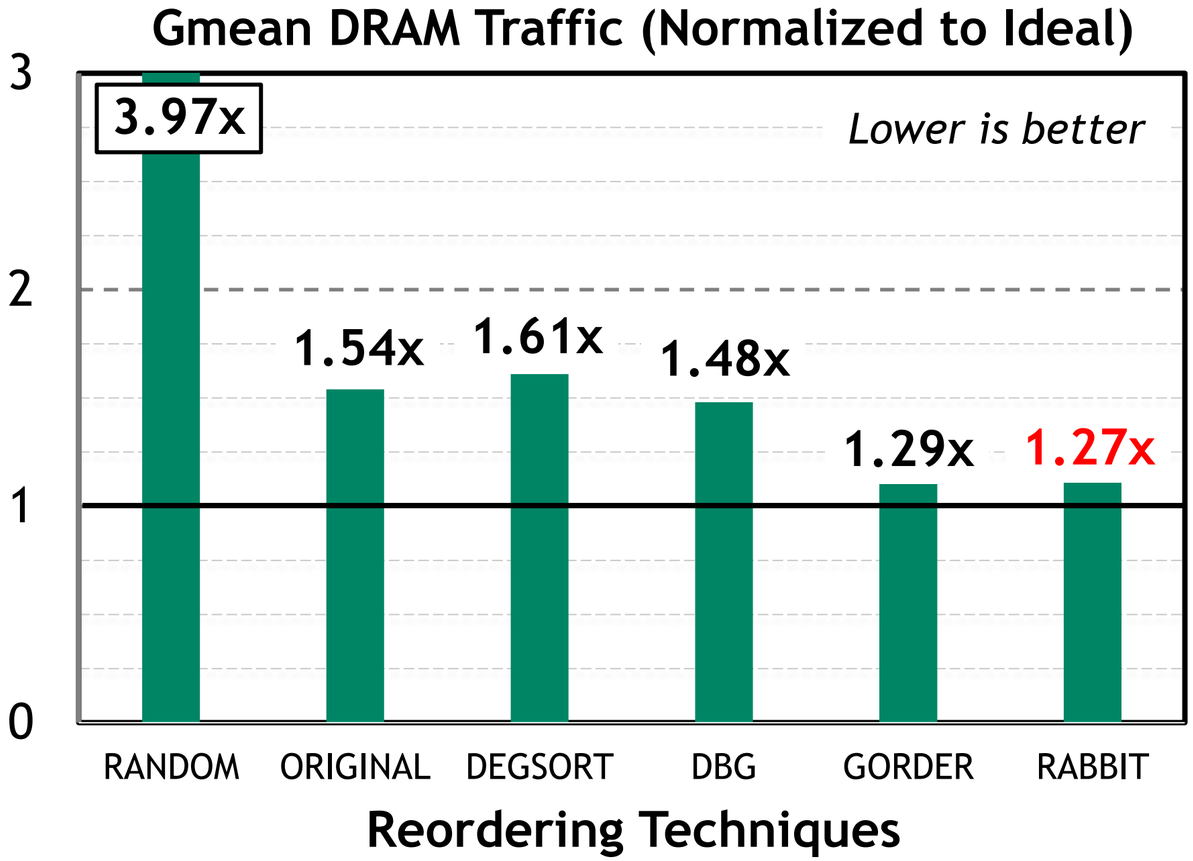

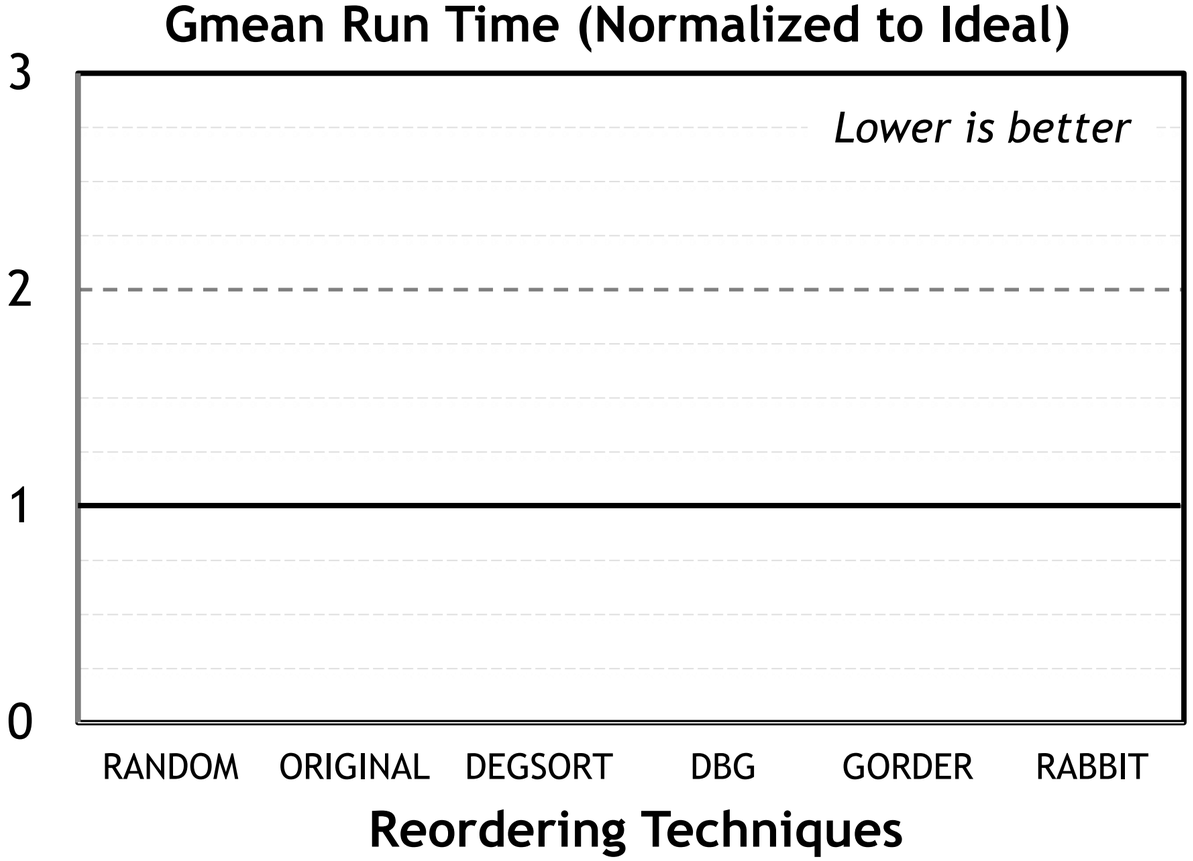

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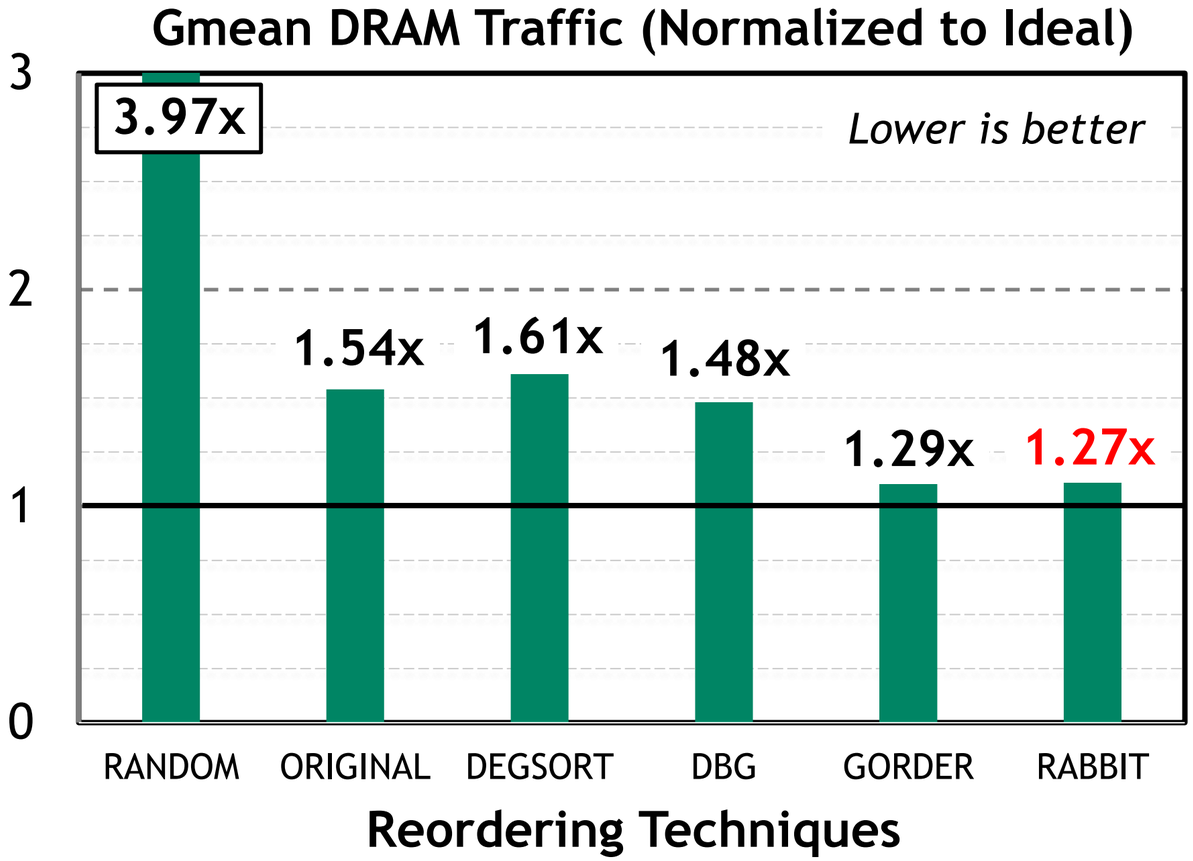

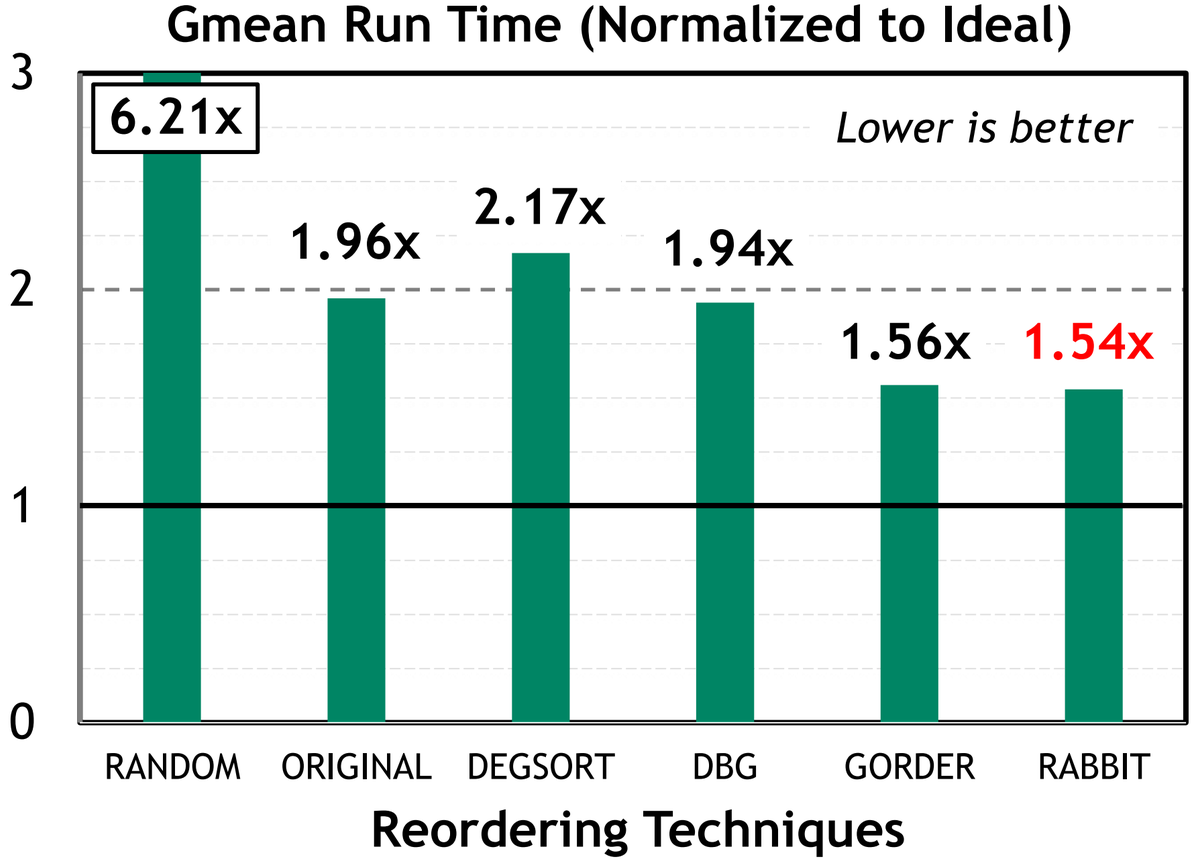

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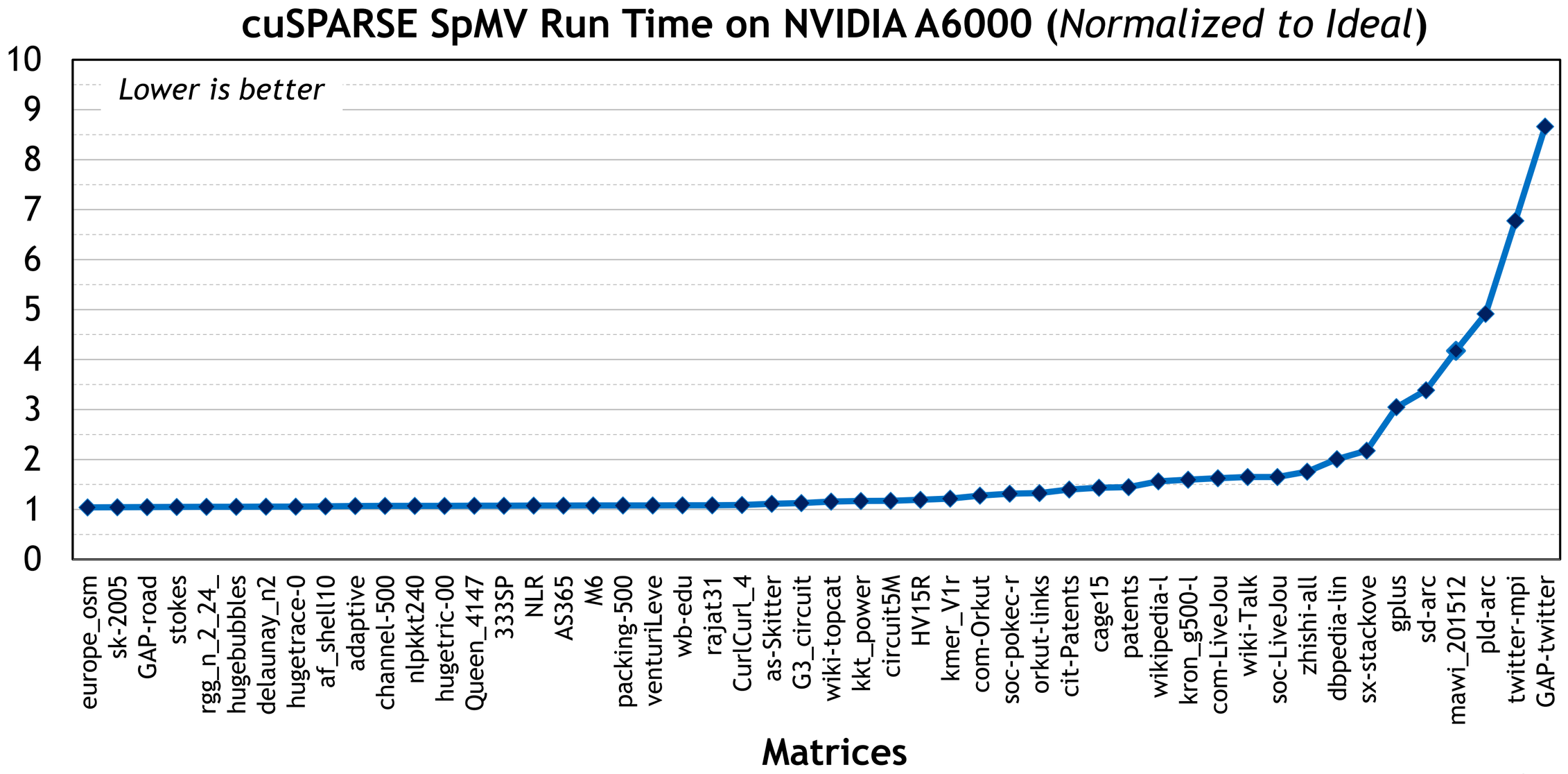

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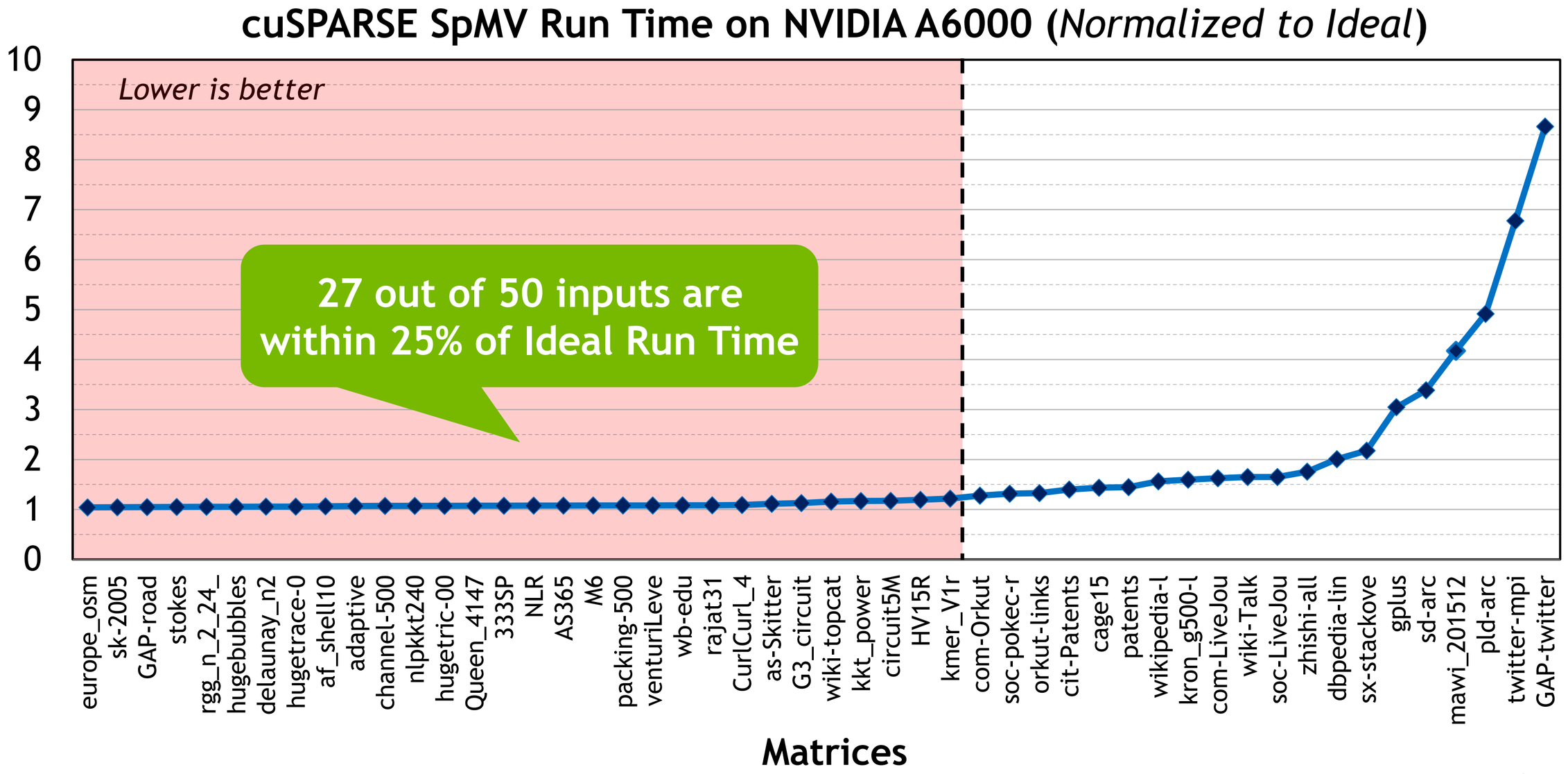

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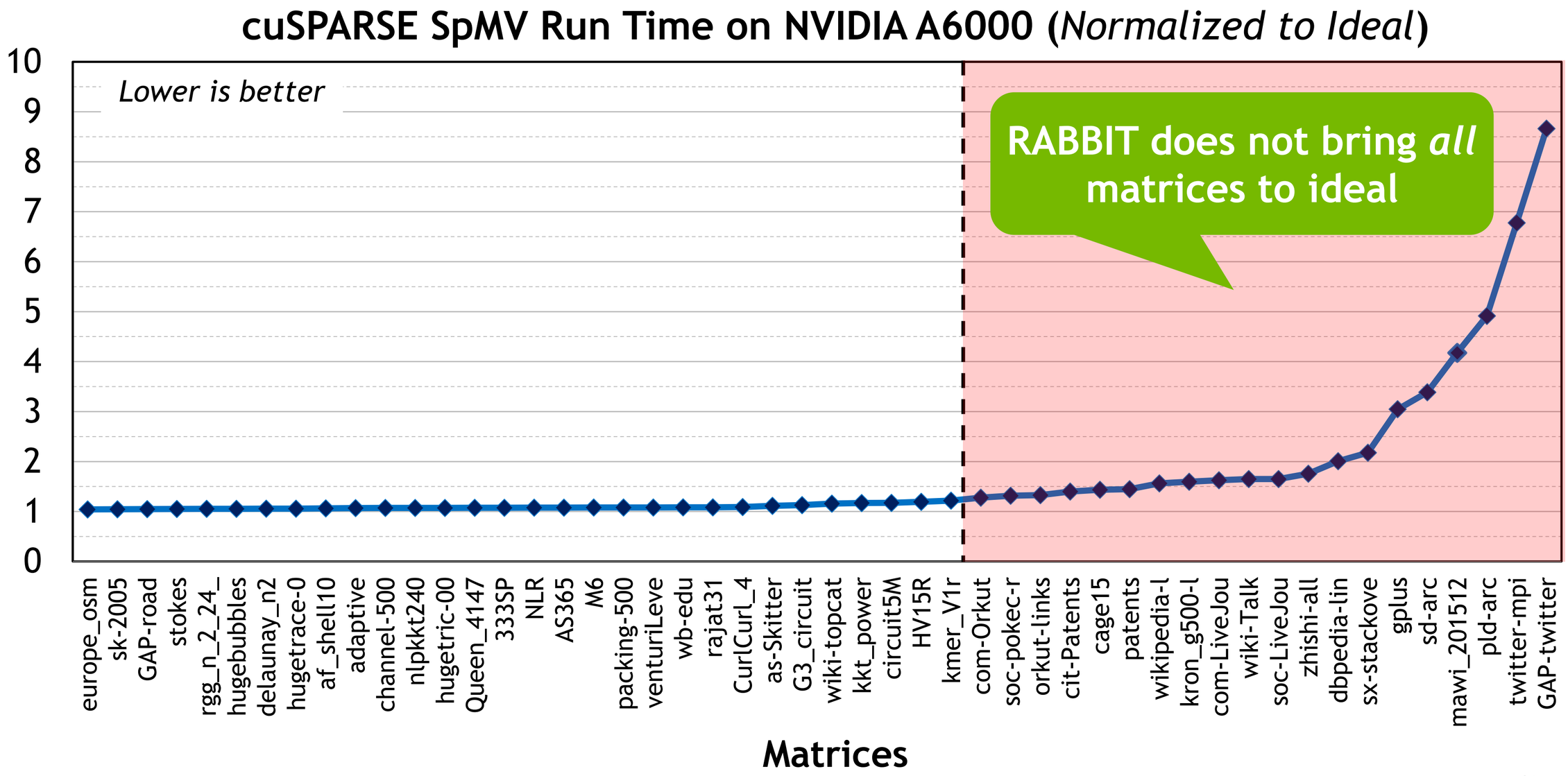

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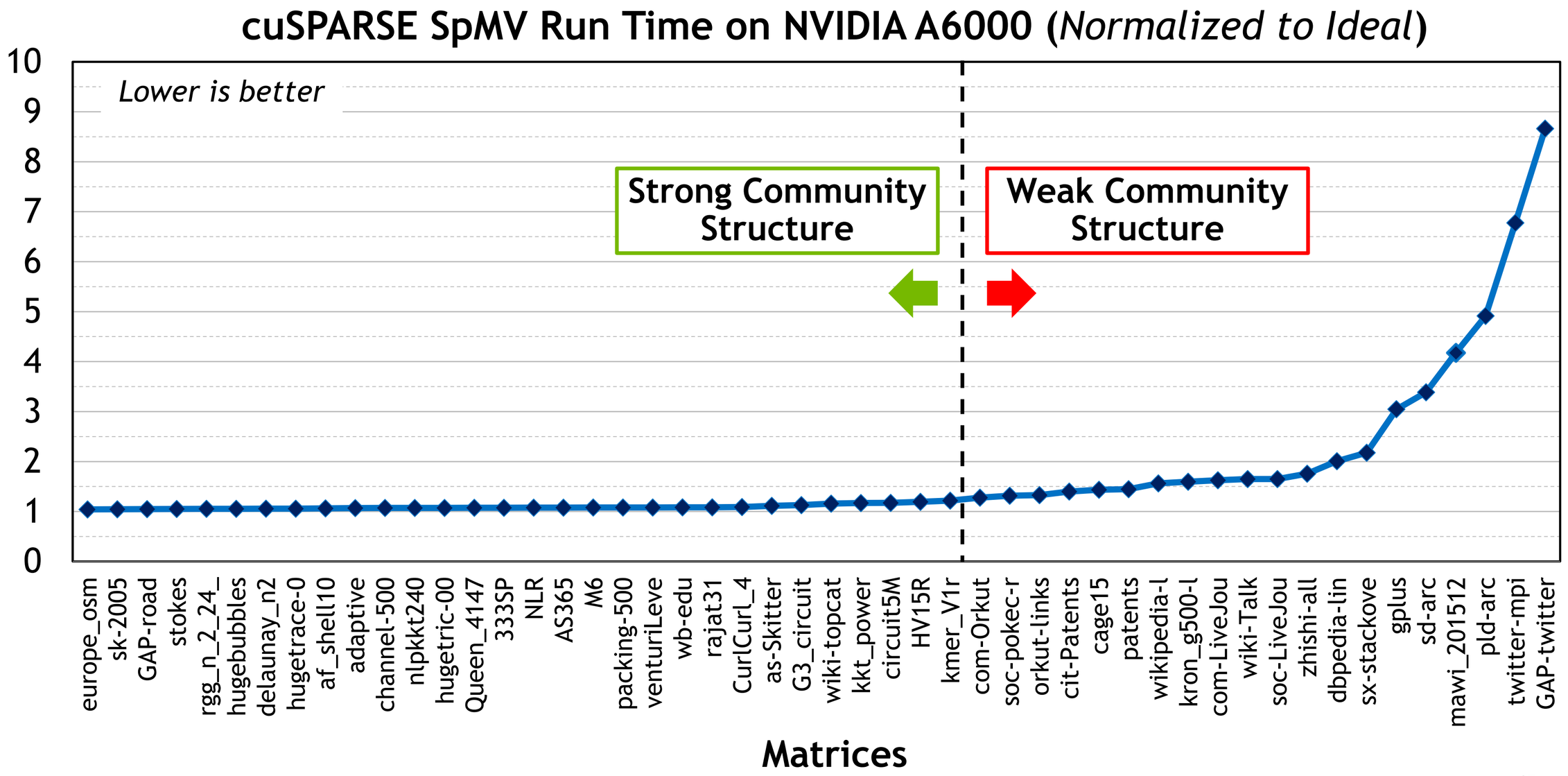
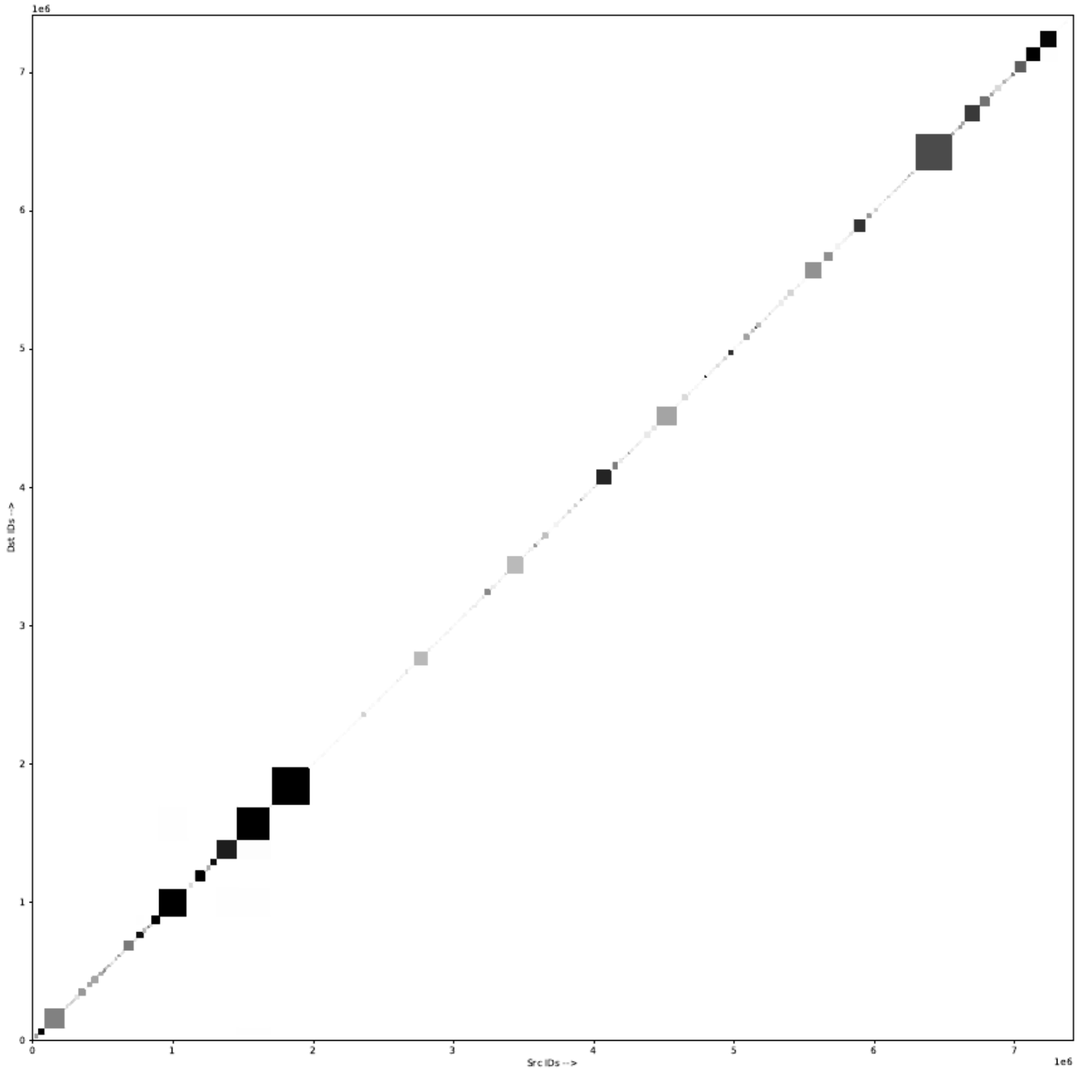


LIMITATIONS OF RABBIT

Strong Community Structure



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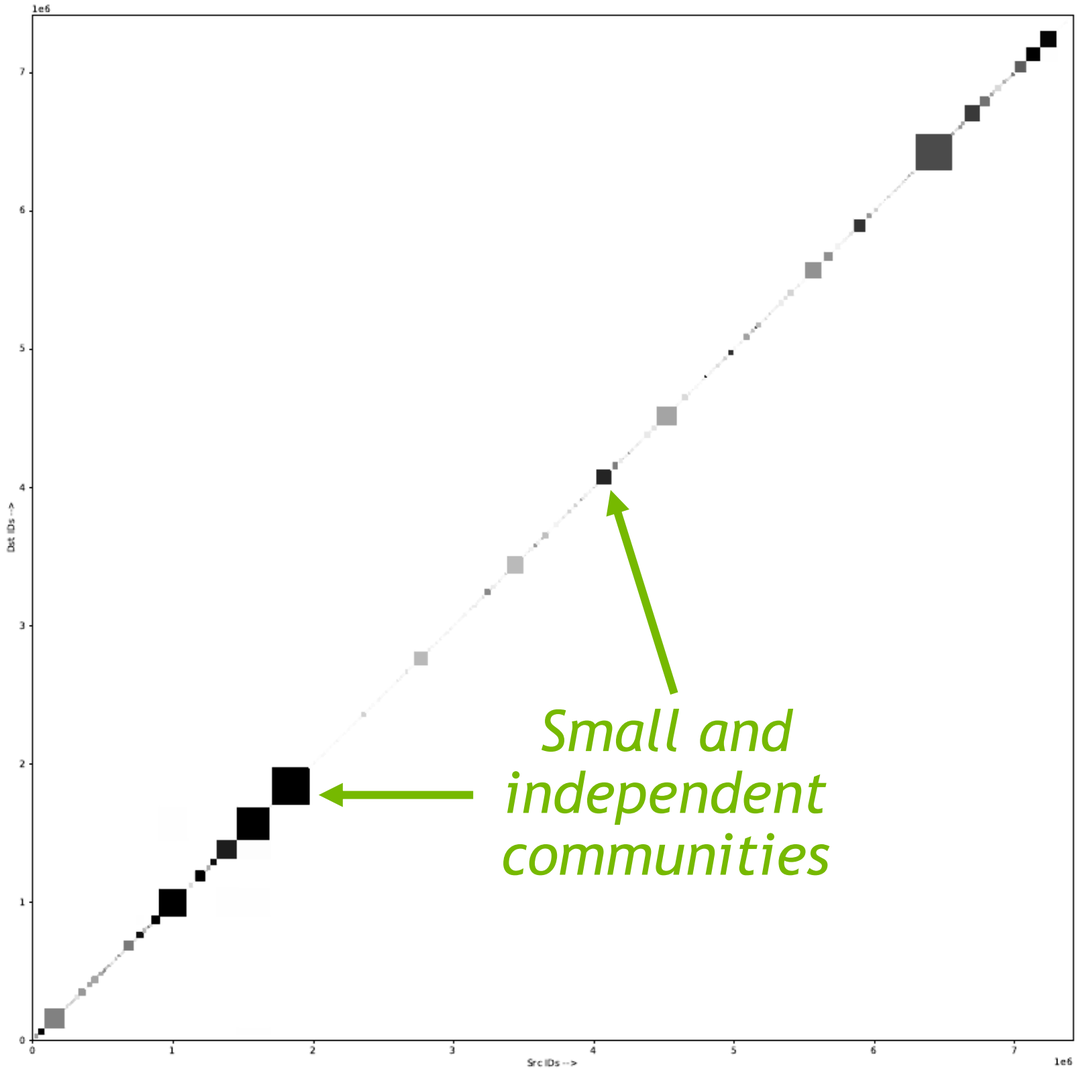

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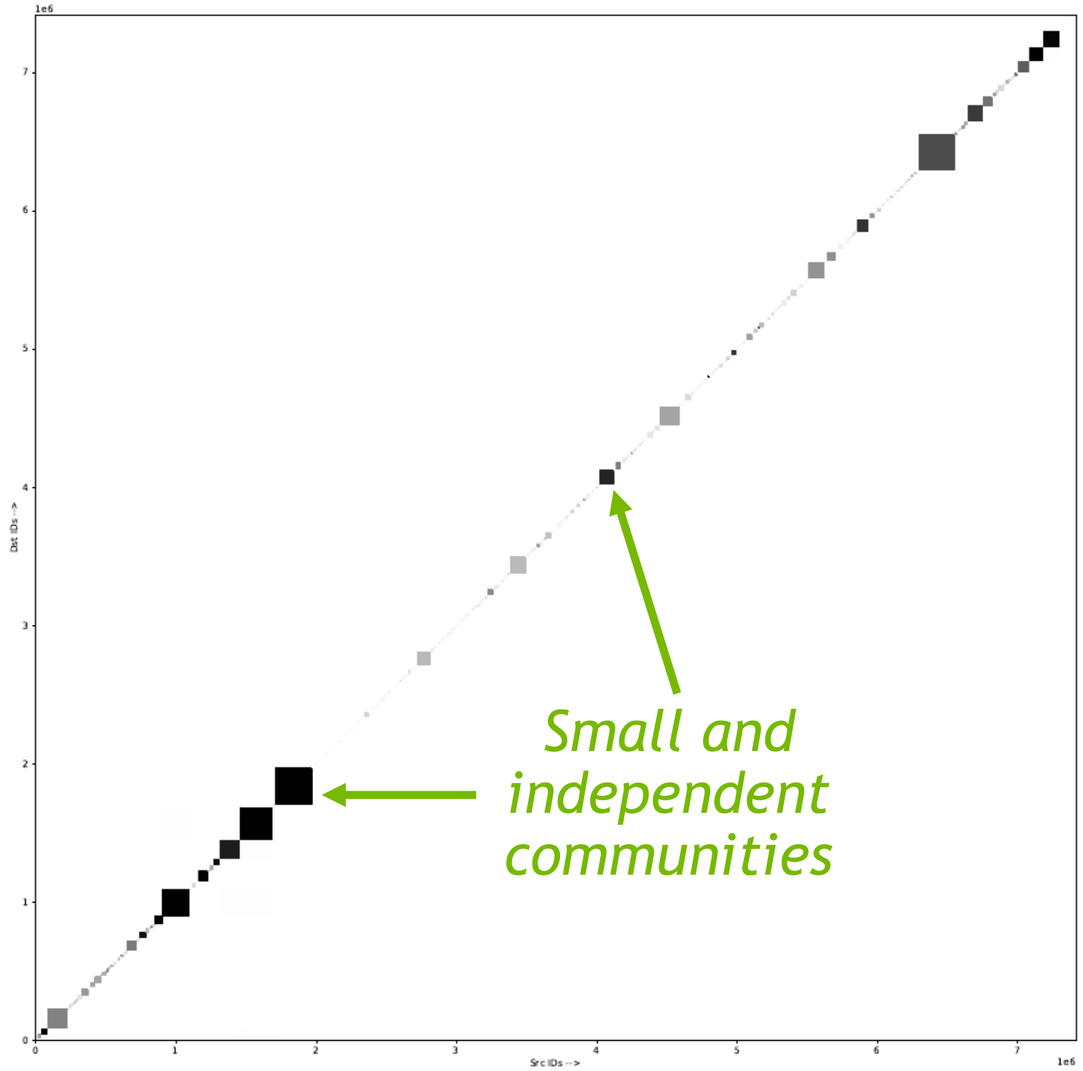

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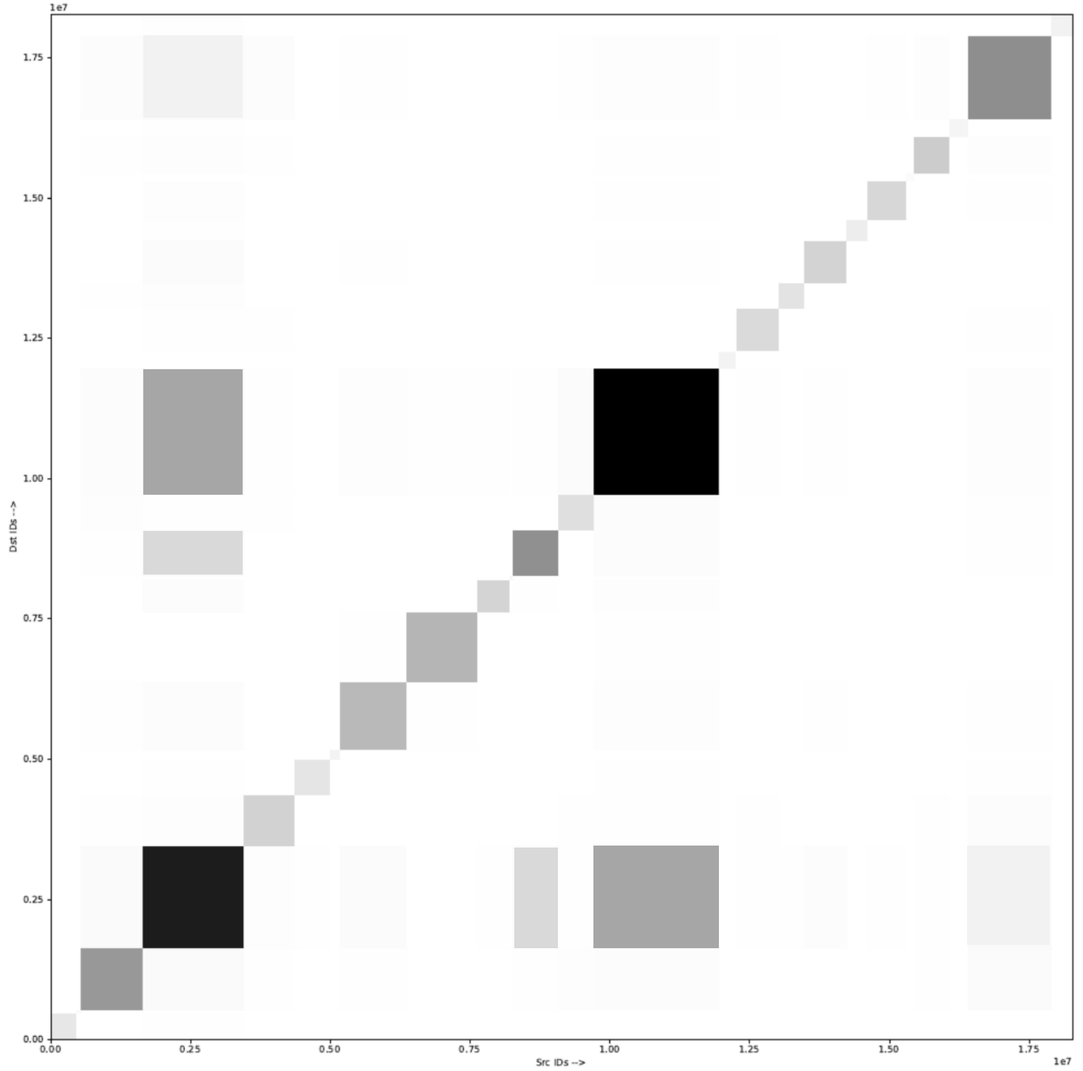

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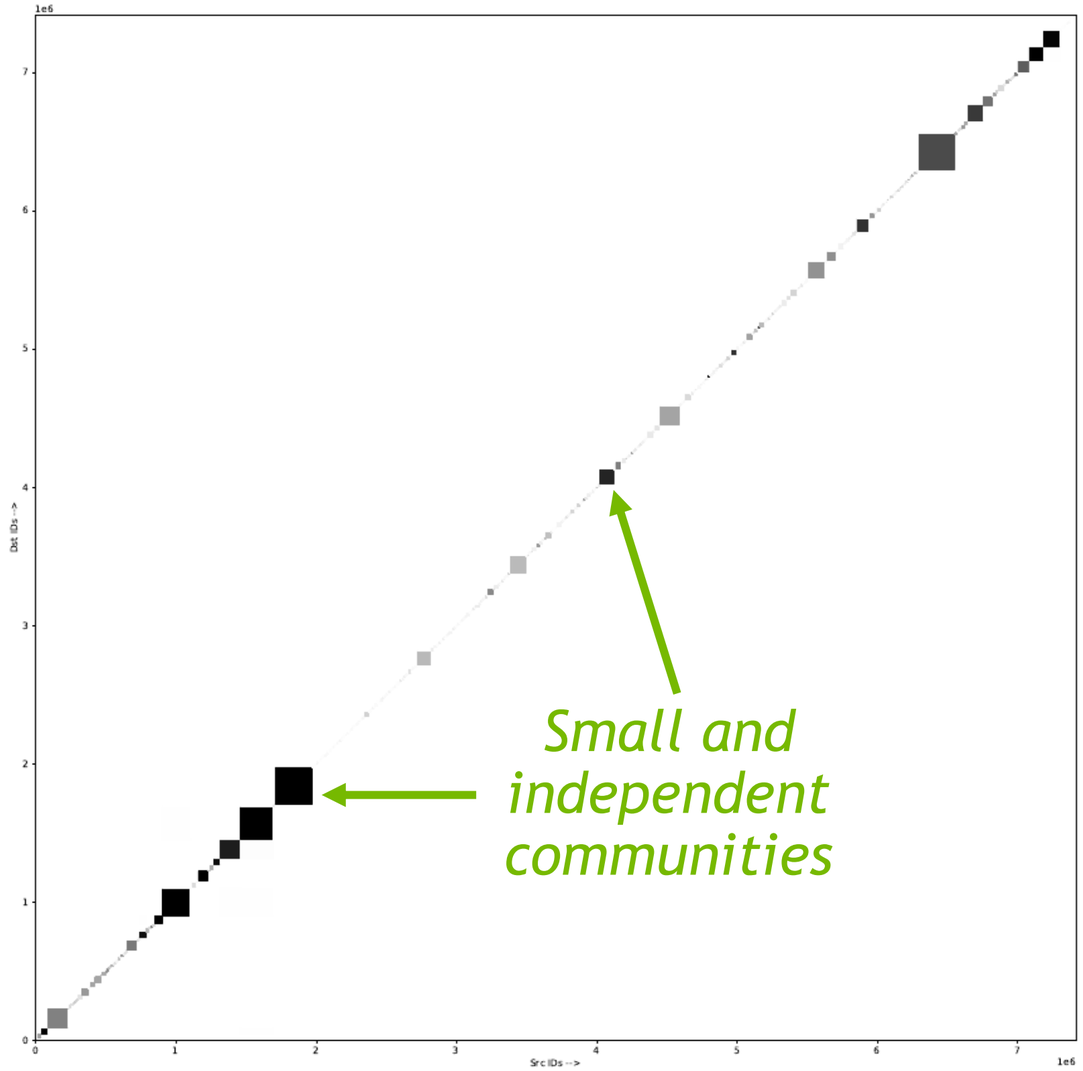

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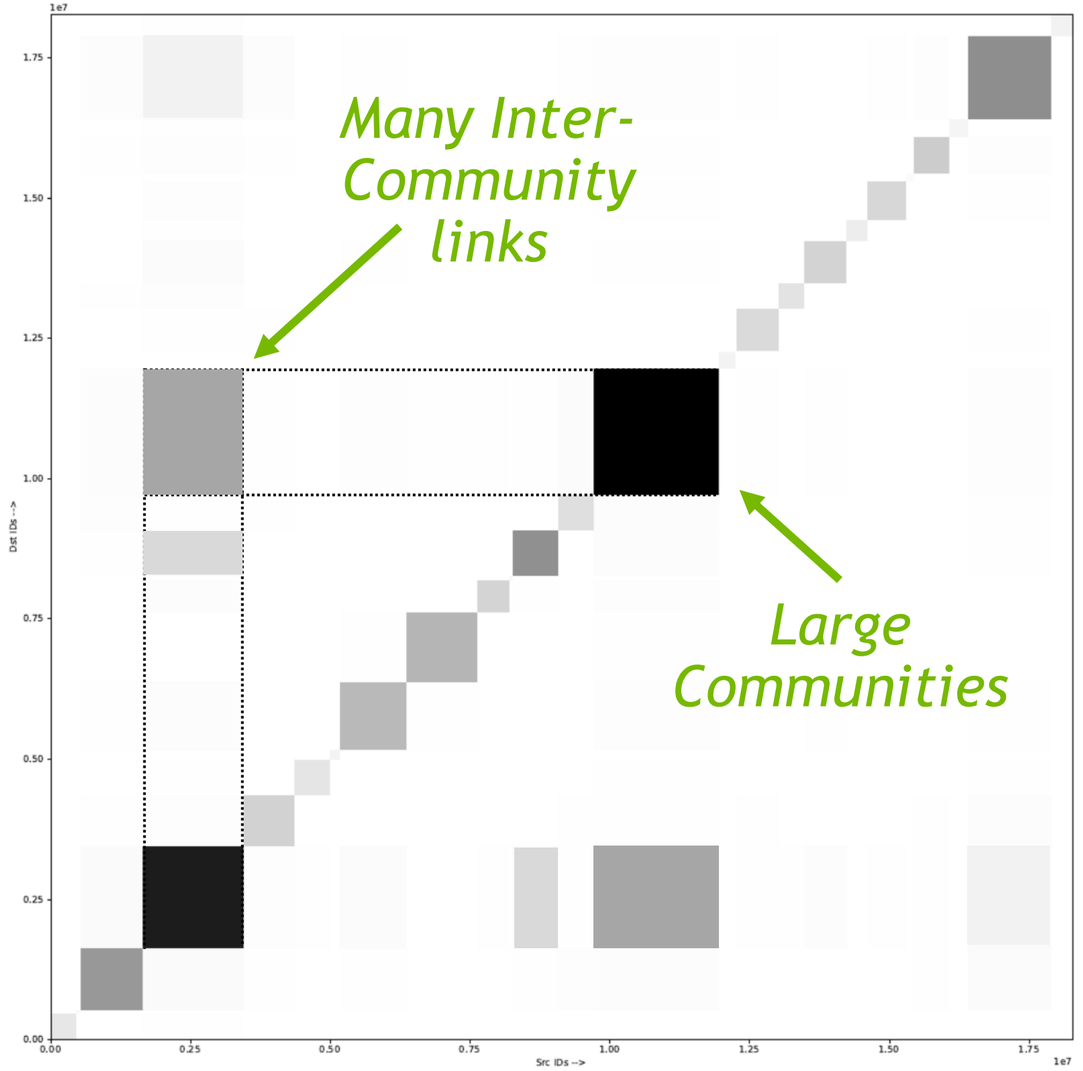
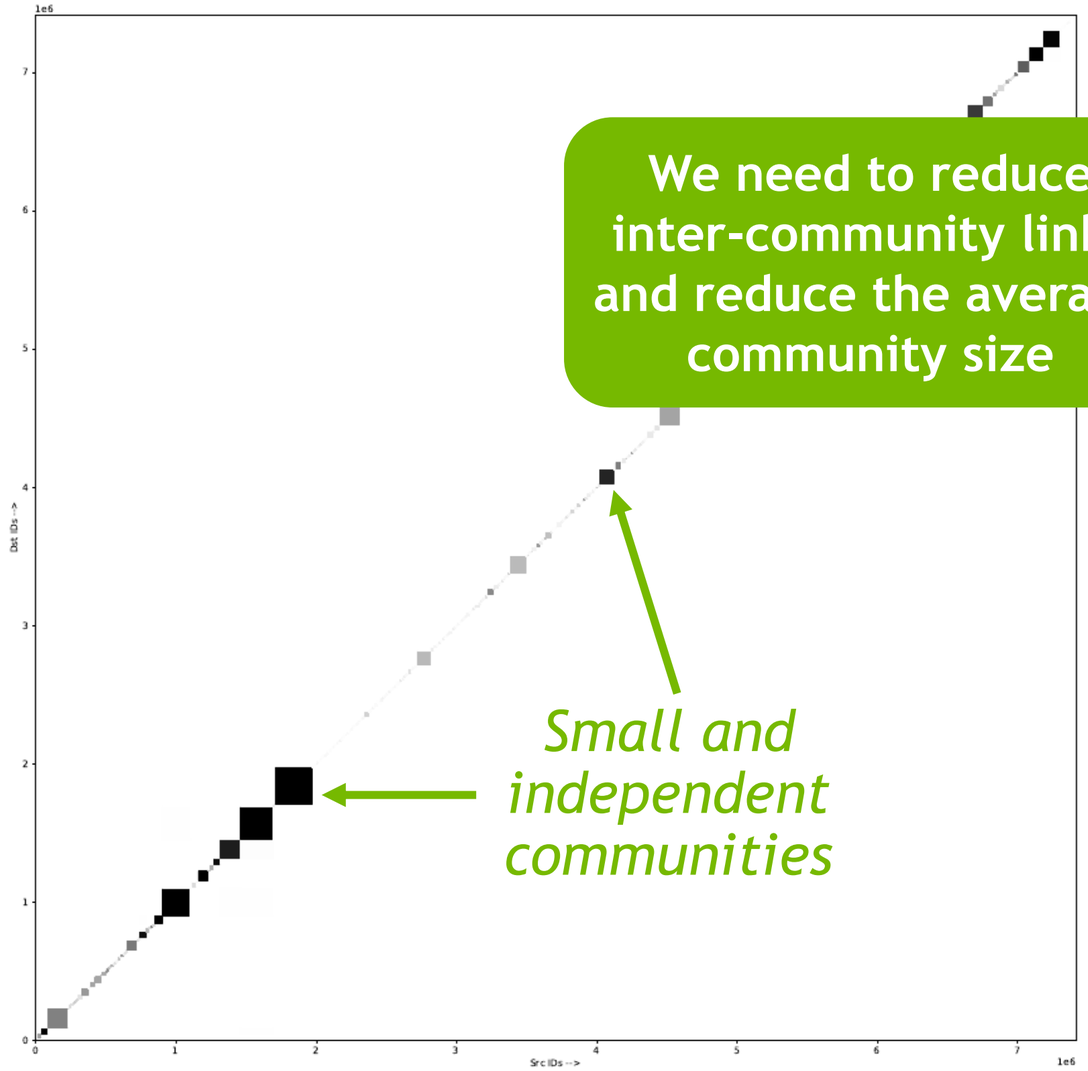


LIMITATIONS OF RABBIT

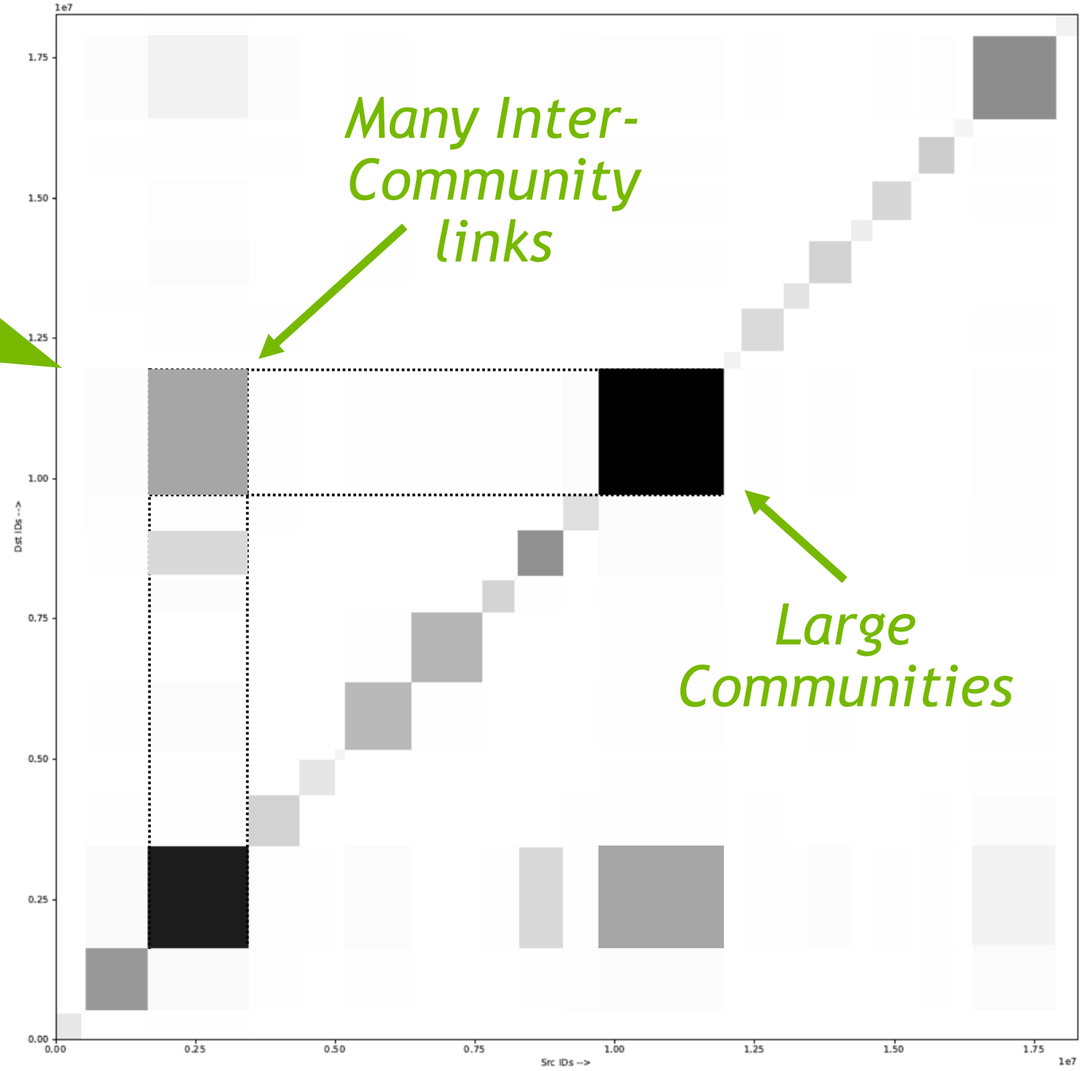
Strong Community Structure



We need to reduce inter-community links and reduce the average community size

Small and independent communities

Weak Community Structure

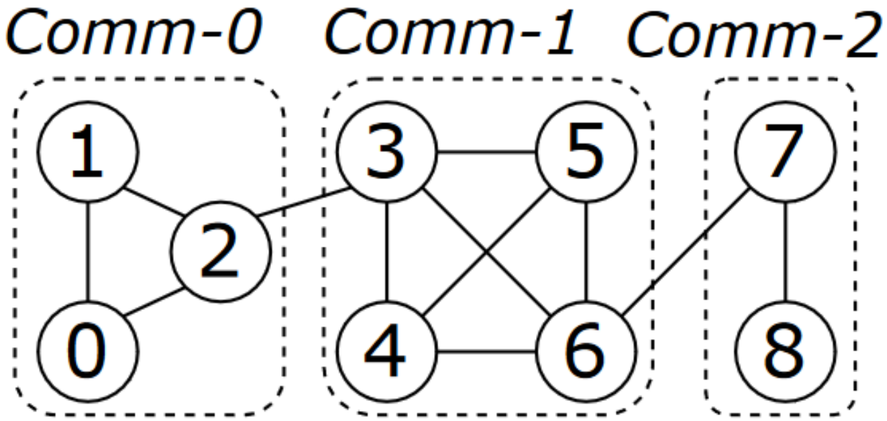


Many Inter-Community links

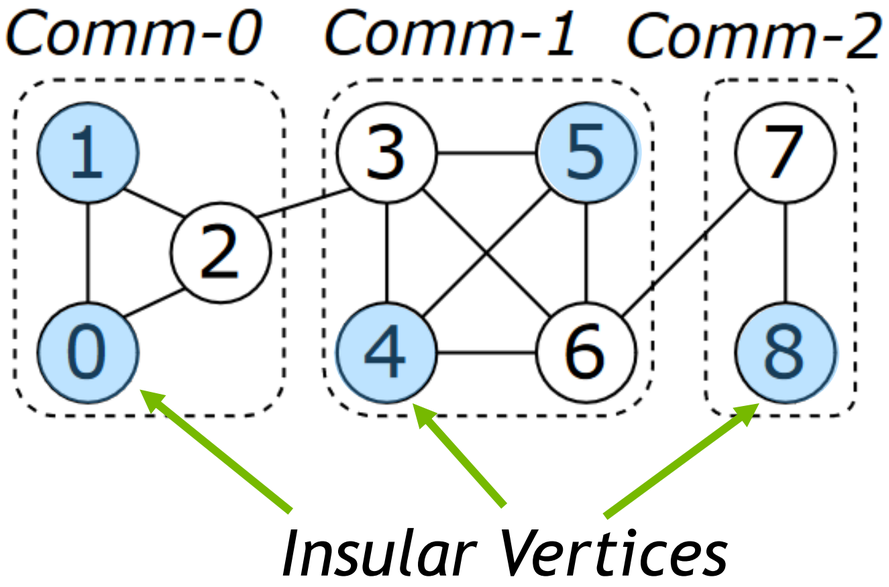
Large Communities

NVIDIA.

PROPERTIES OF INPUTS WITH WEAK COMMUNITY STRUCTURE



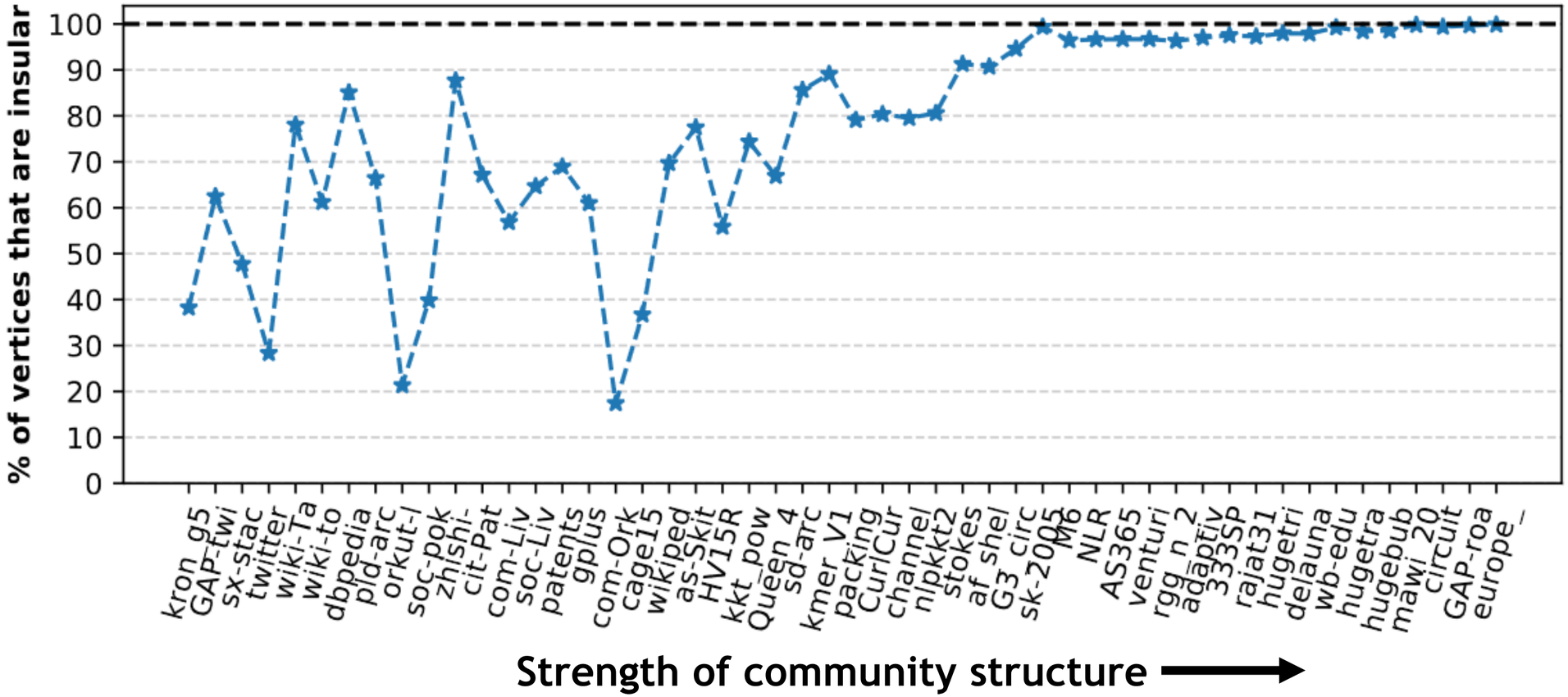
PROPERTIES OF INPUTS WITH WEAK COMMUNITY STRUCTURE



Insular Vertex:
All neighbors are in the same community

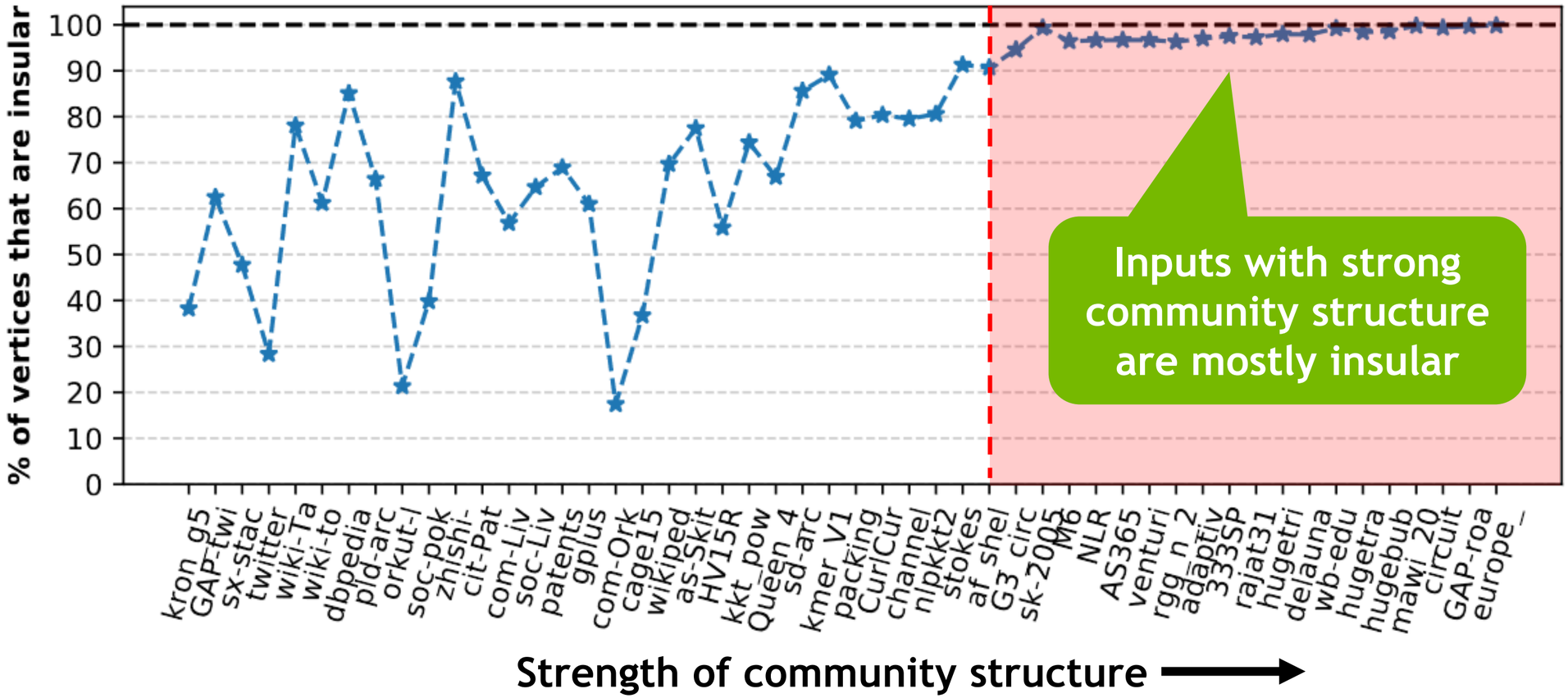
PROPERTIES OF INPUTS WITH WEAK COMMUNITY STRUCTURE

- **PROPERTY #1:** A large percentage of the matrix is ***insular***



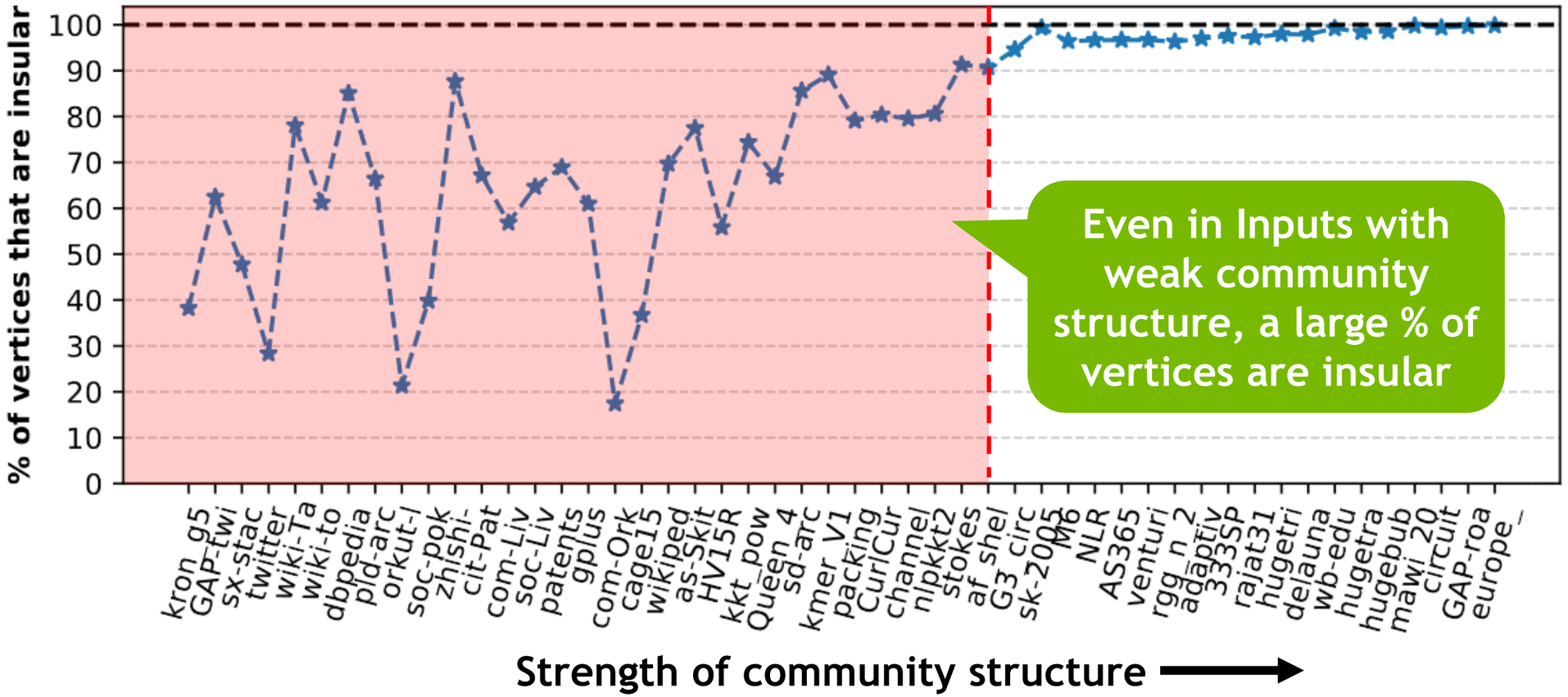
PROPERTIES OF INPUTS WITH WEAK COMMUNITY STRUCTURE

- **PROPERTY #1:** A large percentage of the matrix is ***insular***



PROPERTIES OF INPUTS WITH WEAK COMMUNITY STRUCTURE

- **PROPERTY #1:** A large percentage of the matrix is ***insular***

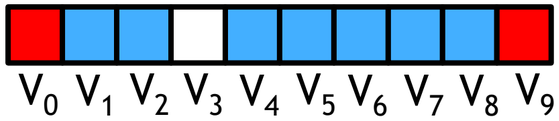


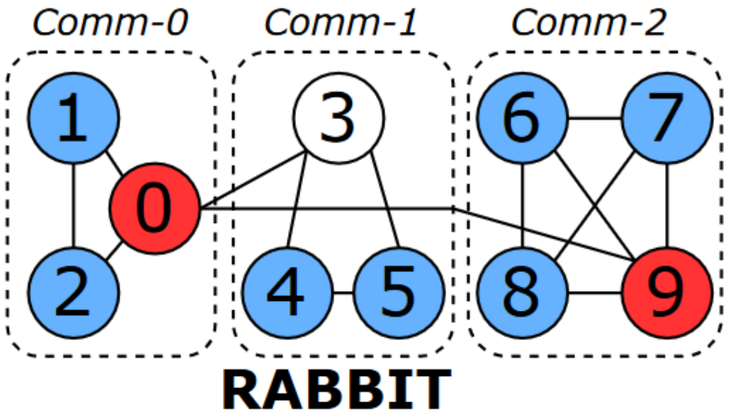
PROPERTIES OF INPUTS WITH WEAK COMMUNITY STRUCTURE

- **PROPERTY #1:** A large percentage of the matrix is ***insular***

- **PROPERTY #2:** ***Hubs*** account for most of the inter-community links
  - *Highly-connected hubs (vertices with degree > average degree) are responsible for 86% for all inter-community links*

ENHANCING RABBIT

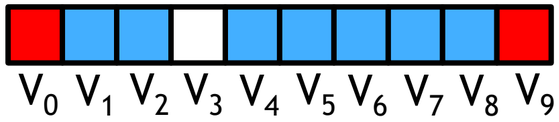


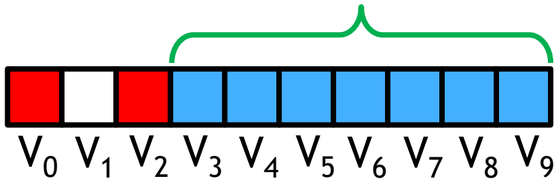


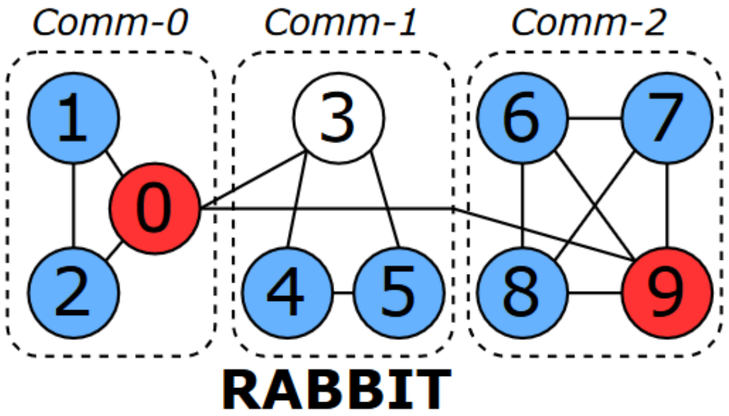
Insular Vertices

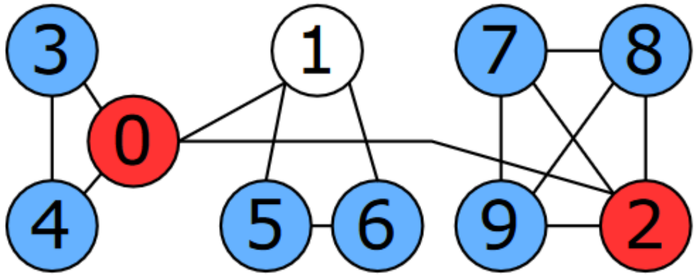
Hub Vertices

ENHANCING RABBIT







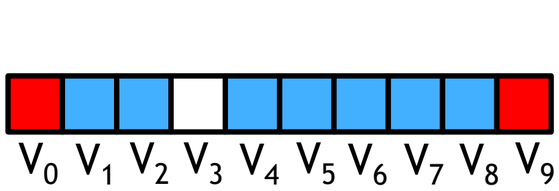


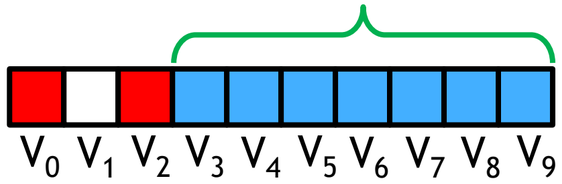
Insular Vertices

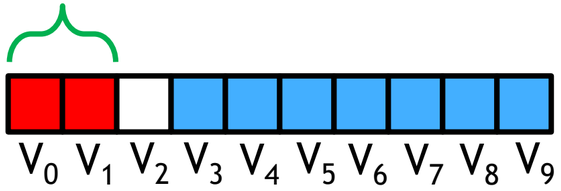
Hub Vertices

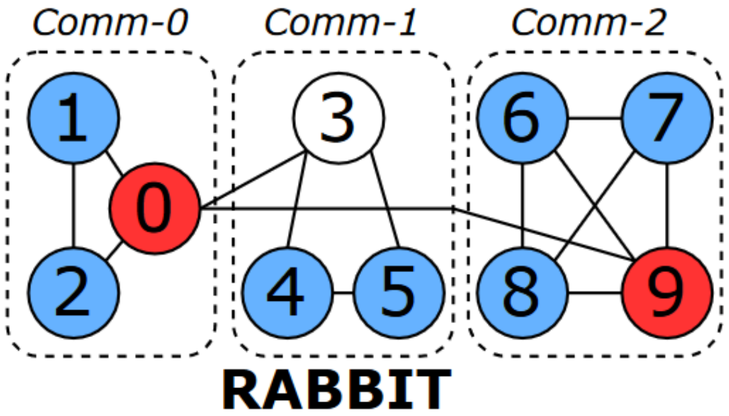
Transformation #1:
Group Insular Vertices

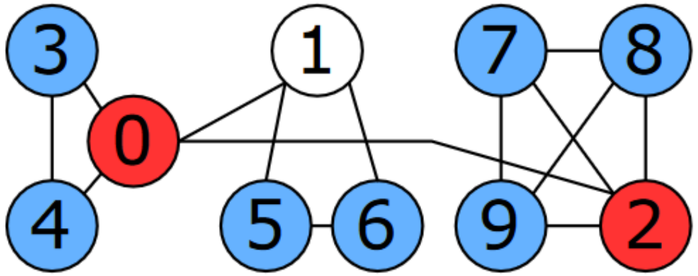
ENHANCING RABBIT

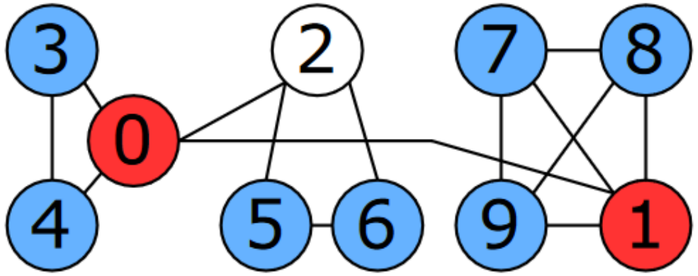












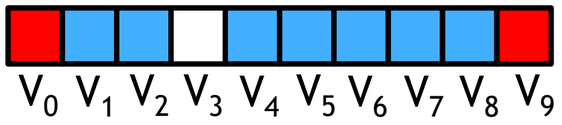
Insular Vertices

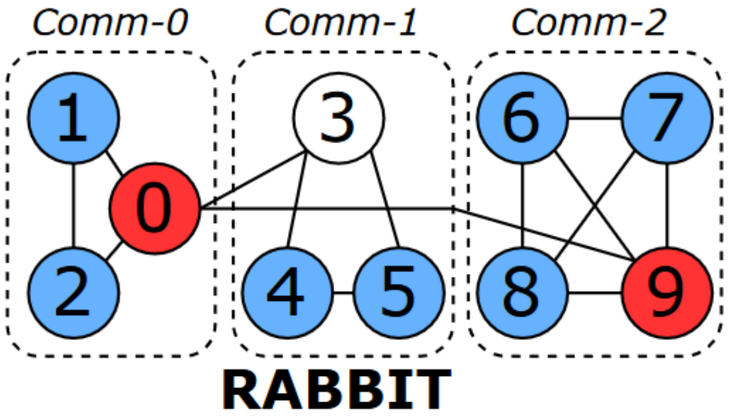
Hub Vertices

Transformation #1:
Group Insular Vertices

Transformation #2:
Group Hub Vertices

ENHANCING RABBIT

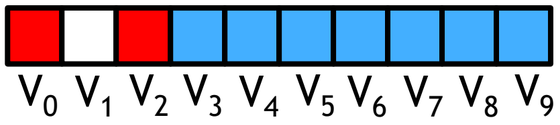


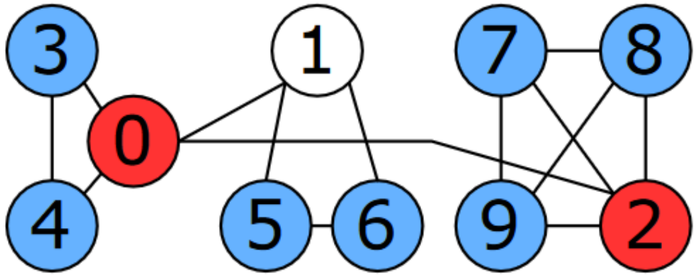


RABBIT

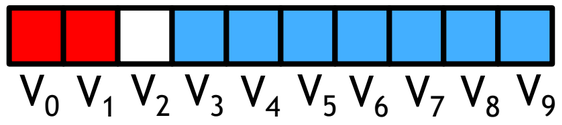
Insular Vertices

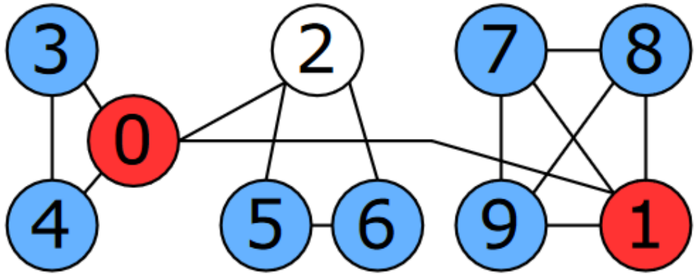
Hub Vertices





Transformation #1:
Group Insular Vertices



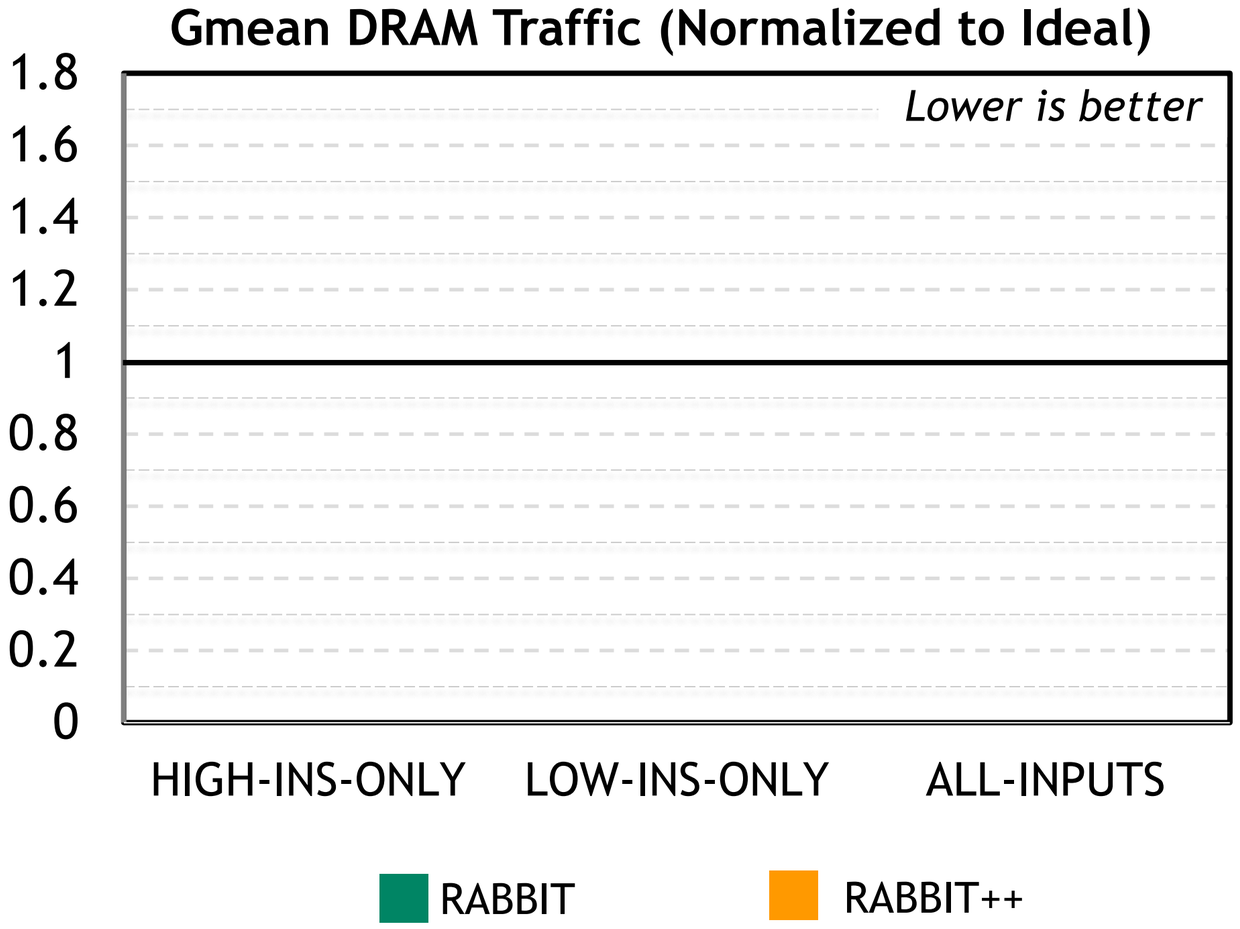


Transformation #2:
Group Hub Vertices

RABBIT++

PERFORMANCE IMPROVEMENTS WITH RABBIT++

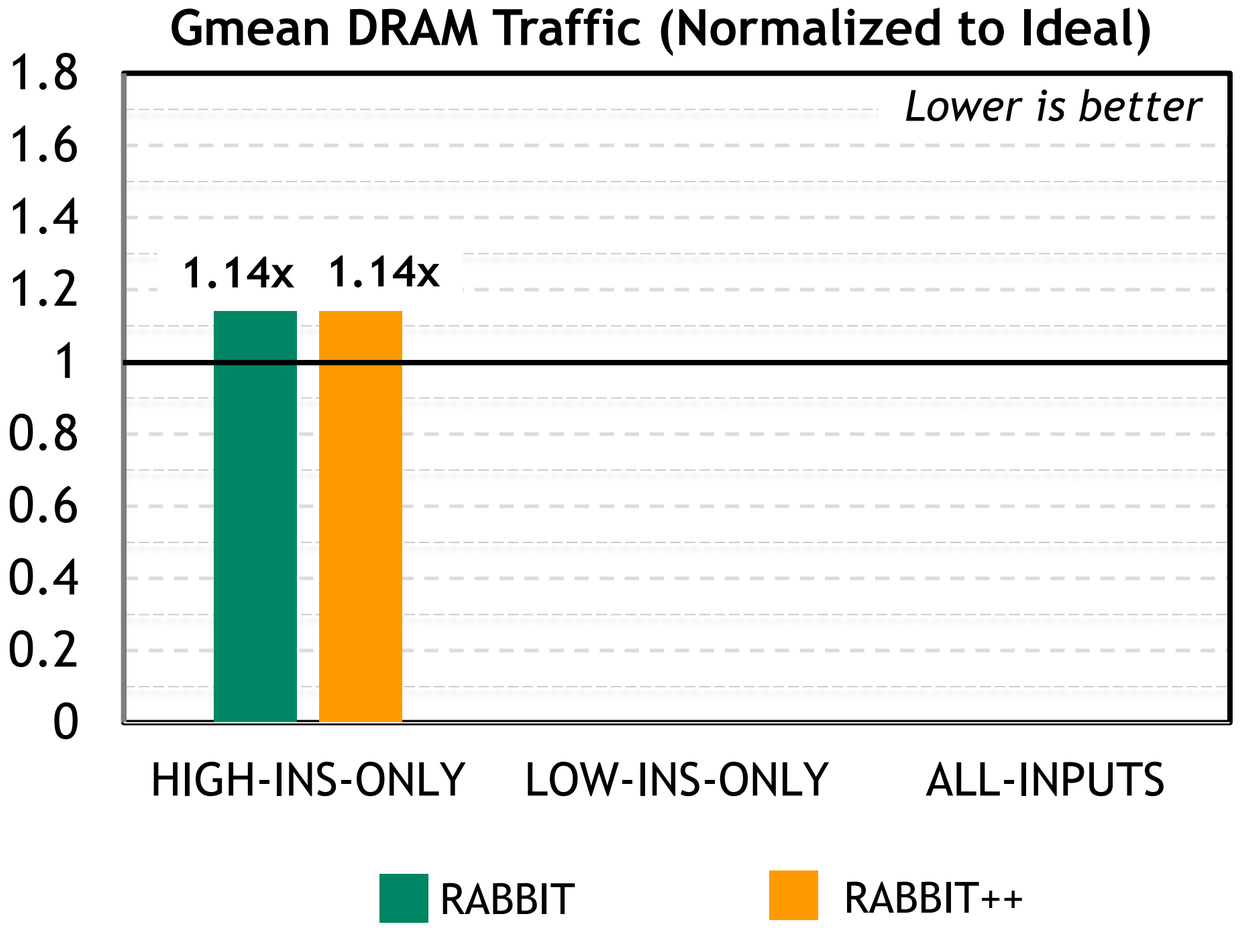
cuSPARSE SpMV on NVIDIA A6000 GPU



Ideal Traffic = Algorithmic Minimum DRAM Transfers

PERFORMANCE IMPROVEMENTS WITH RABBIT++

cuSPARSE SpMV on NVIDIA A6000 GPU



Ideal Traffic = Algorithmic Minimum DRAM Transfers

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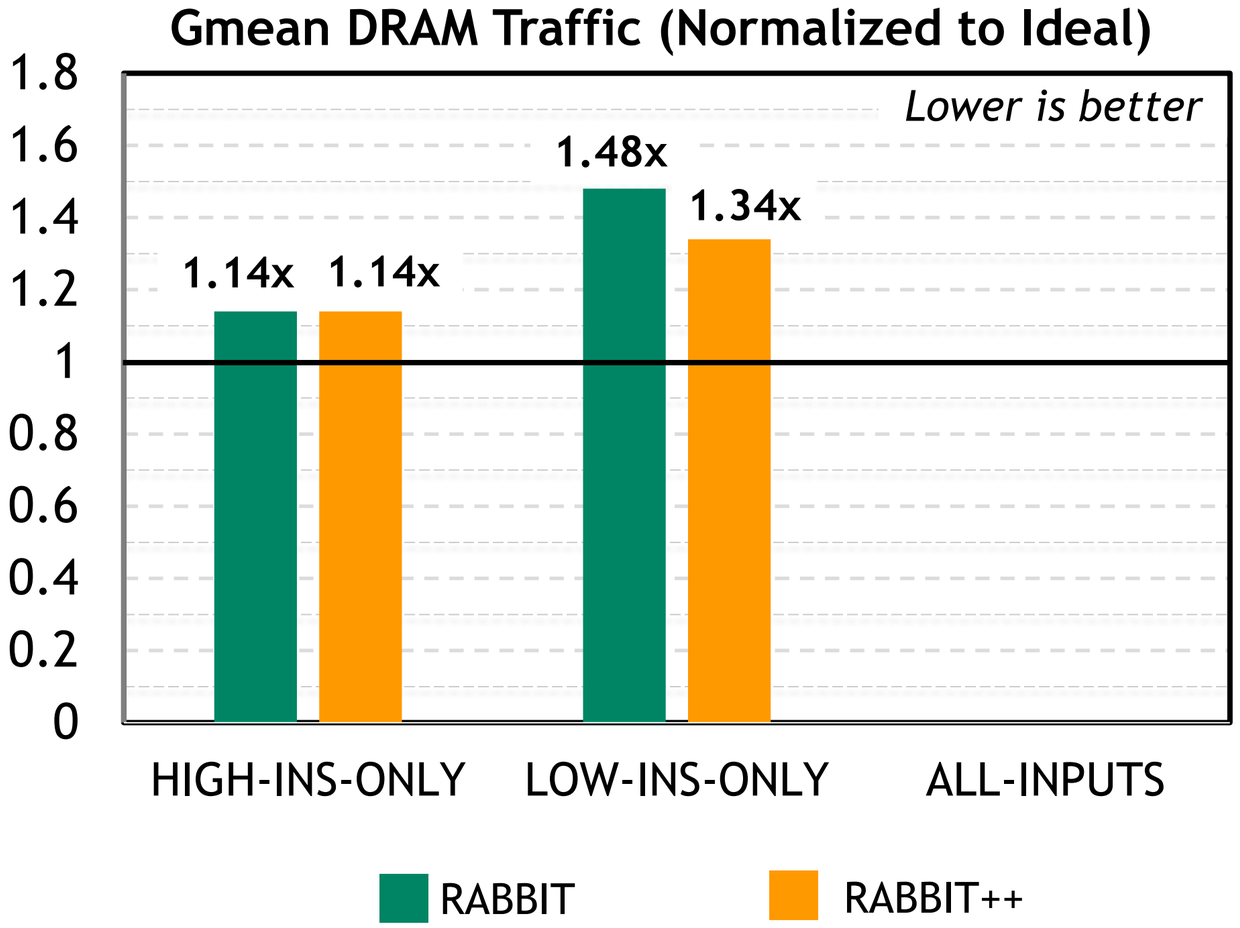
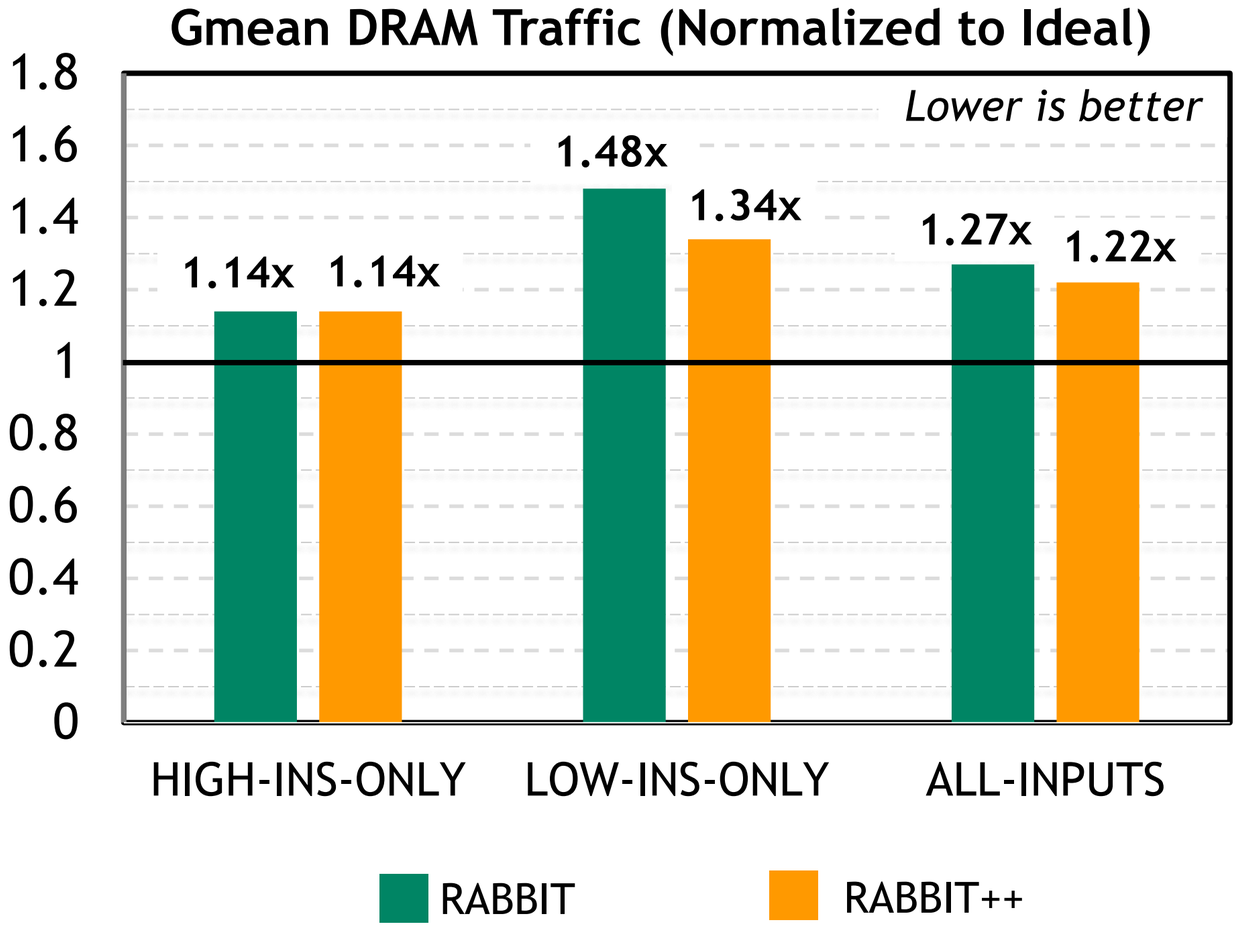
cuSPARSE SpMV on NVIDIA A6000 GPU



Ideal Traffic = Algorithmic Minimum DRAM Transfers

PERFORMANCE IMPROVEMENTS WITH RABBIT++

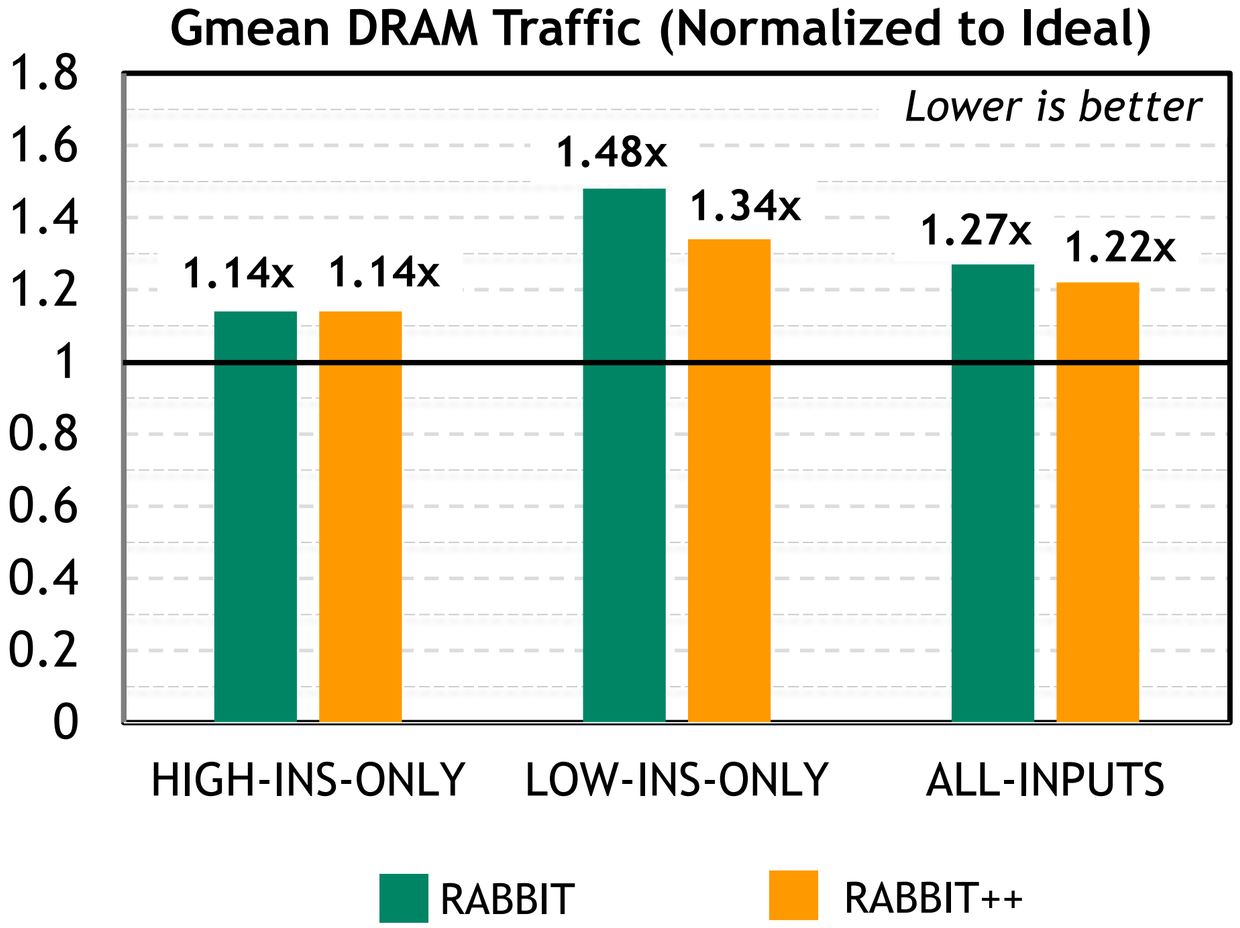
cuSPARSE SpMV on NVIDIA A6000 GPU

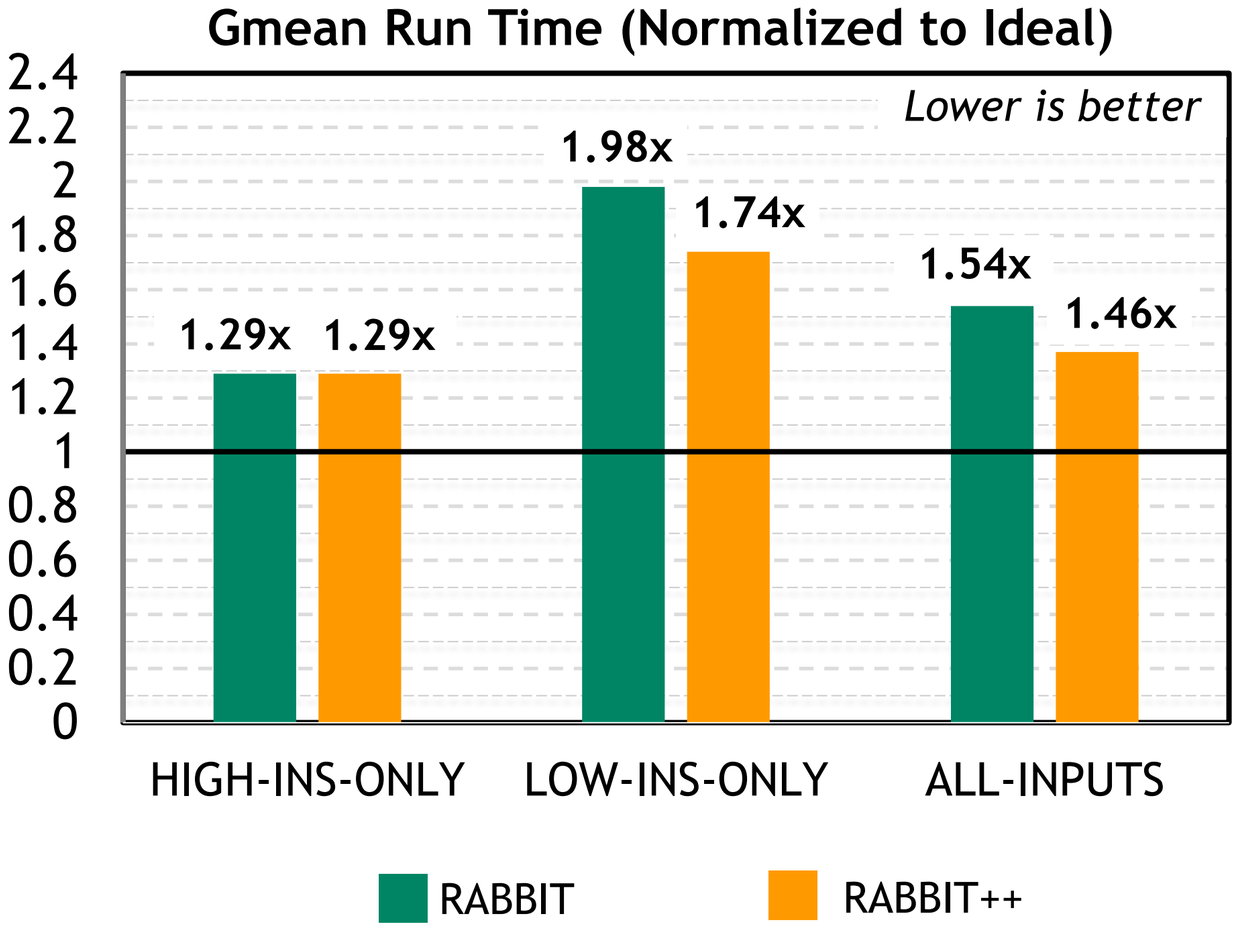


Ideal Traffic = Algorithmic Minimum DRAM Transfers

PERFORMANCE IMPROVEMENTS WITH RABBIT++

cuSPARSE SpMV on NVIDIA A6000 GPU



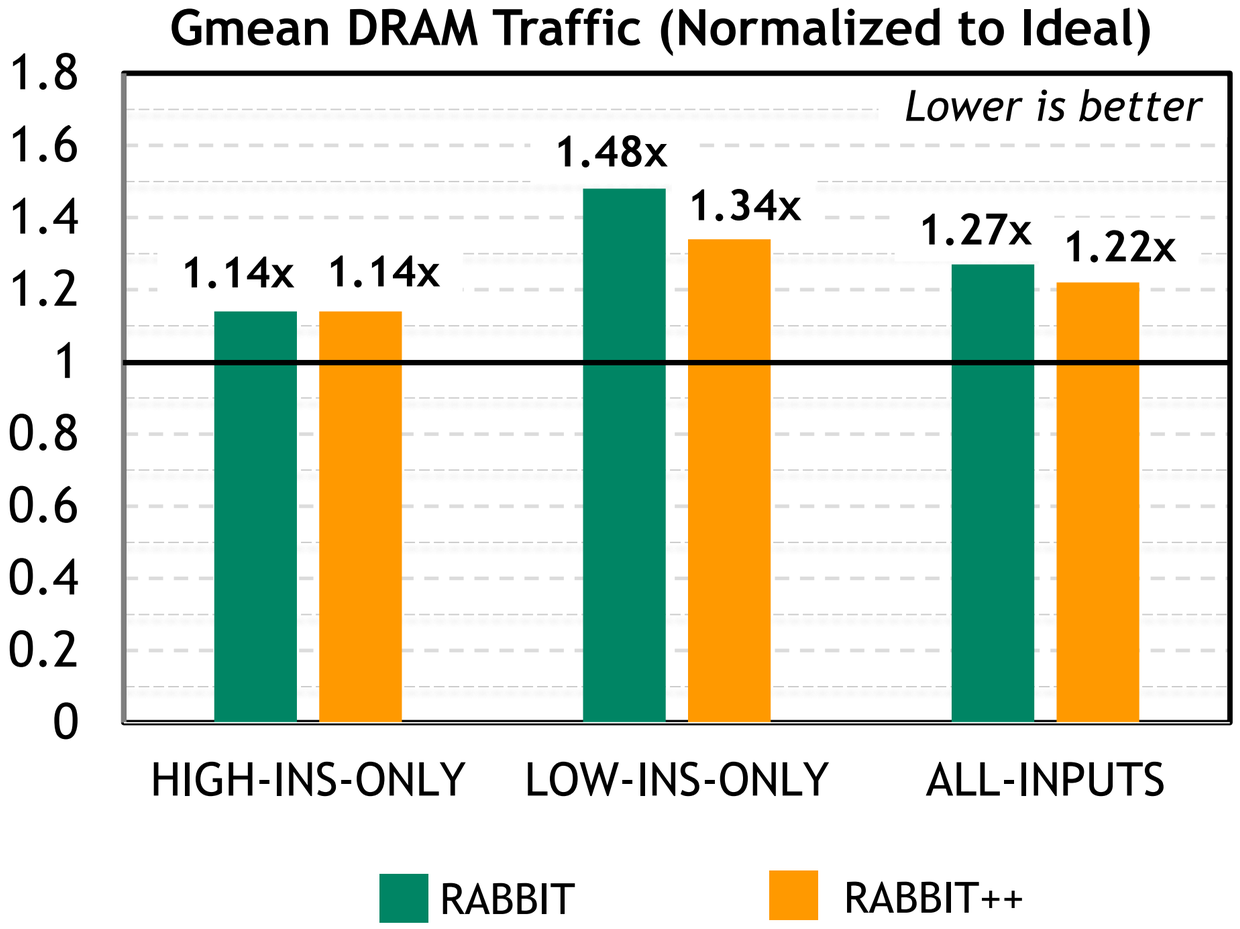


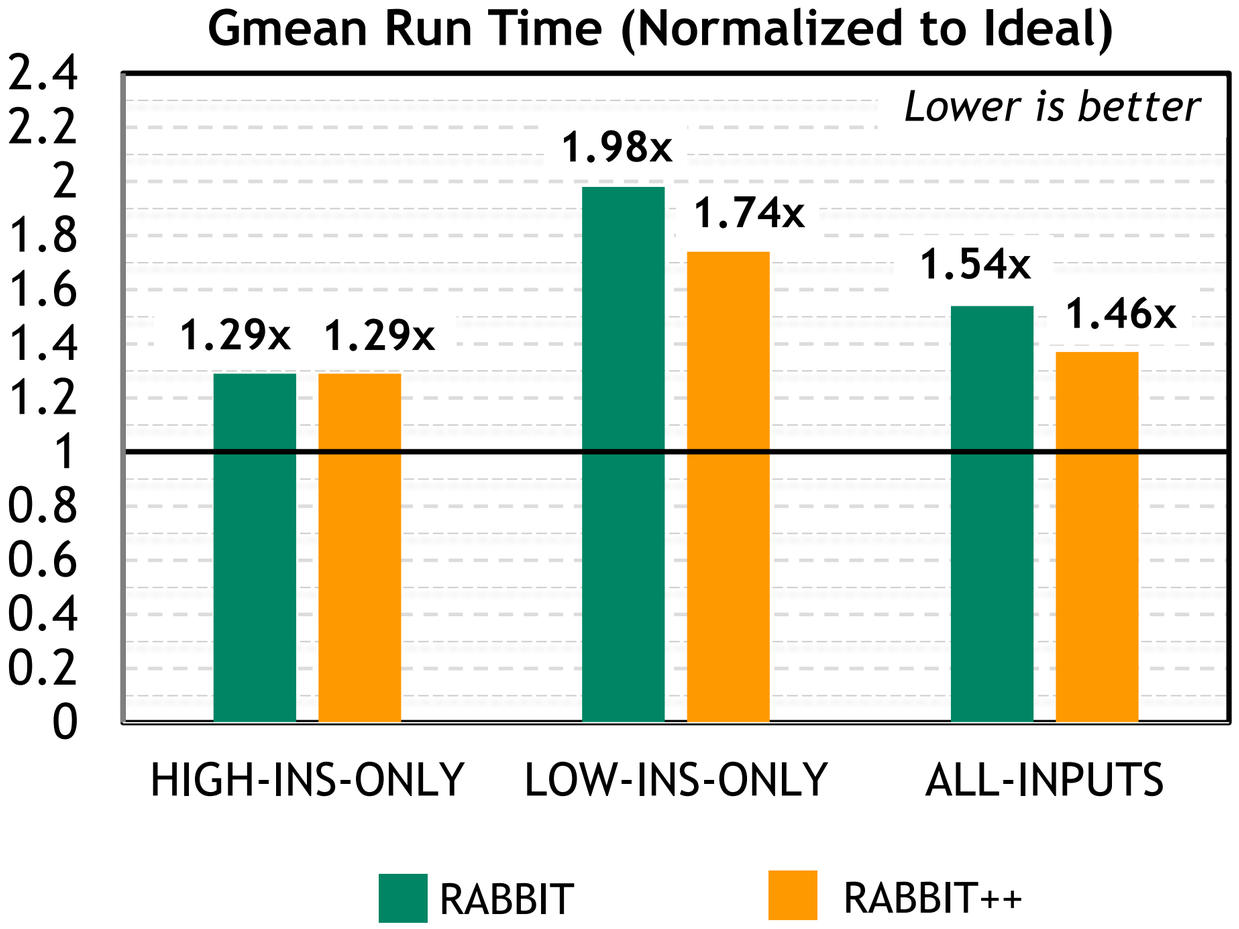
Ideal Traffic = Algorithmic Minimum DRAM Transfers

$$\text{Ideal Run Time} = \frac{\text{Algorithmic Min Traffic}}{\text{Peak DRAM Bandwidth}}$$

PERFORMANCE IMPROVEMENTS WITH RABBIT++

cuSPARSE SpMV on NVIDIA A6000 GPU





RABBIT++ improves SpMV performance over RABBIT by up to 1.6x

MORE DETAILS IN THE PAPER

- RABBIT++ offers DRAM traffic reductions close to Belady's Optimal cache replacement policy
- RABBIT++ brings multiple compressed representations and kernels closest to hardware limits
- Preprocessing overheads of RABBIT(++)

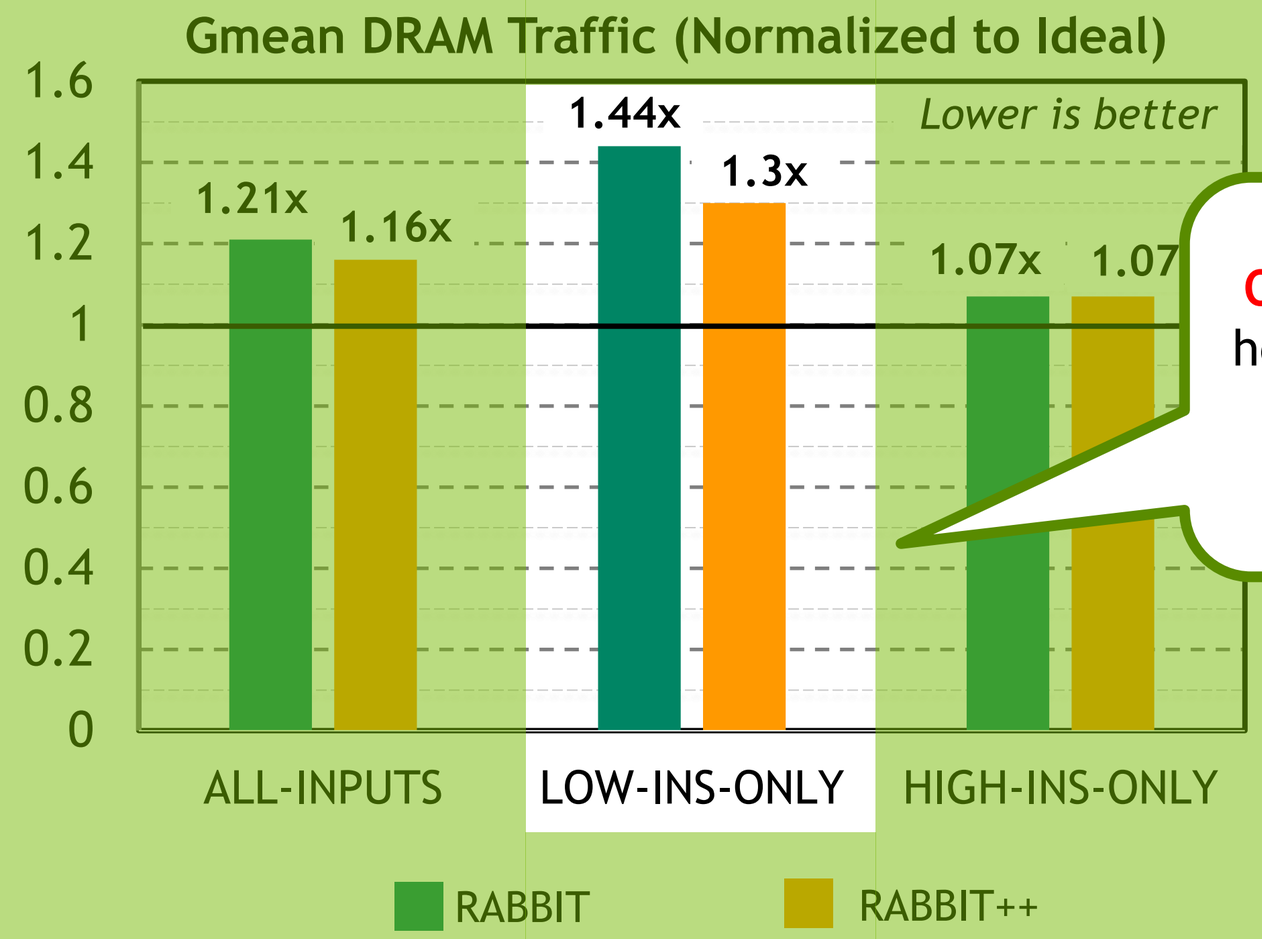
COMMUNITY-BASED MATRIX REORDERING FOR SPARSE LINEAR ALGEBRA OPTIMIZATION

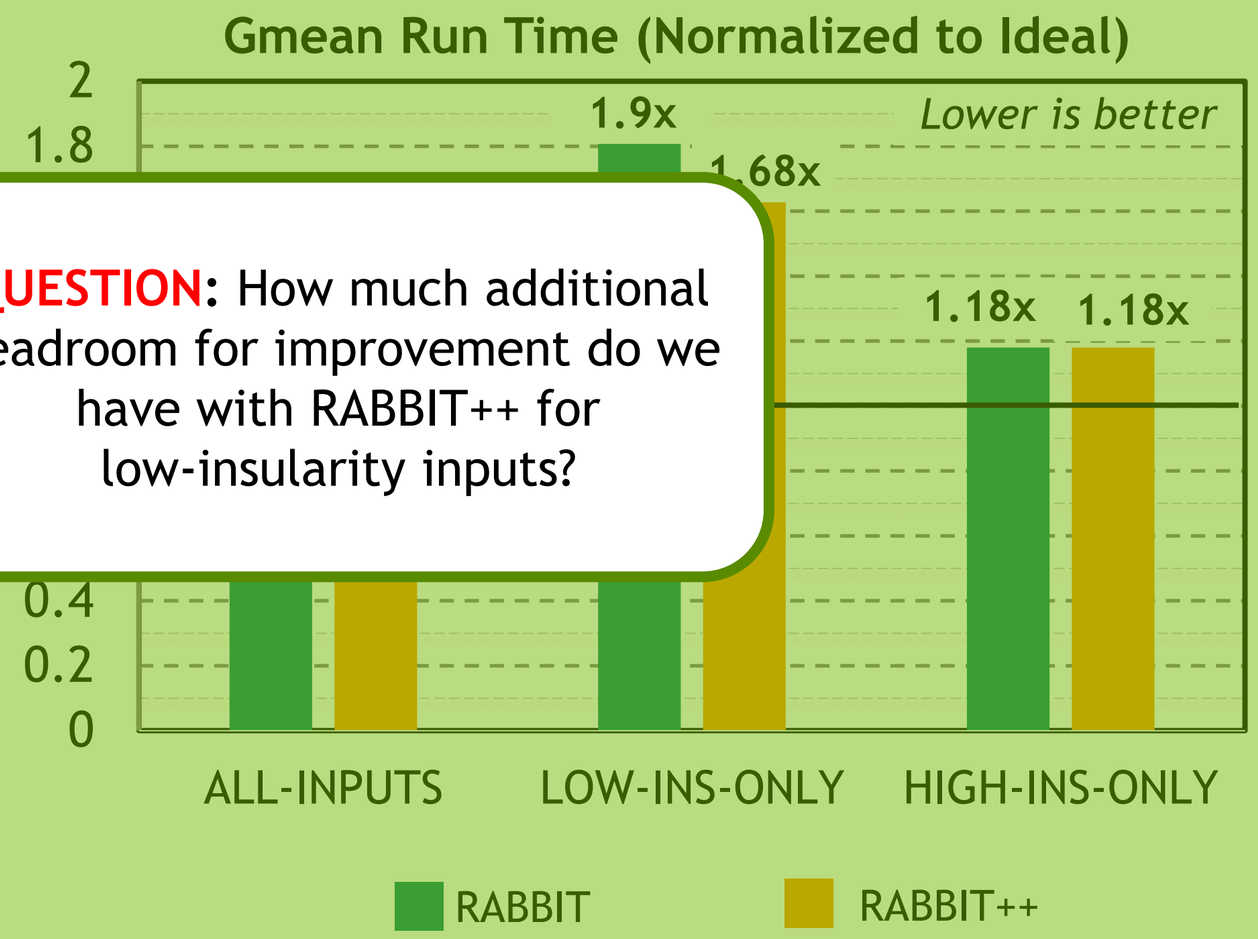
Vignesh Balaji Neal Crago Aamer Jaleel Stephen W. Keckler

BACKUP SLIDES

PERFORMANCE IMPROVEMENTS WITH RABBIT++

cuSPARSE SpMV on NVIDIA A6000 GPU



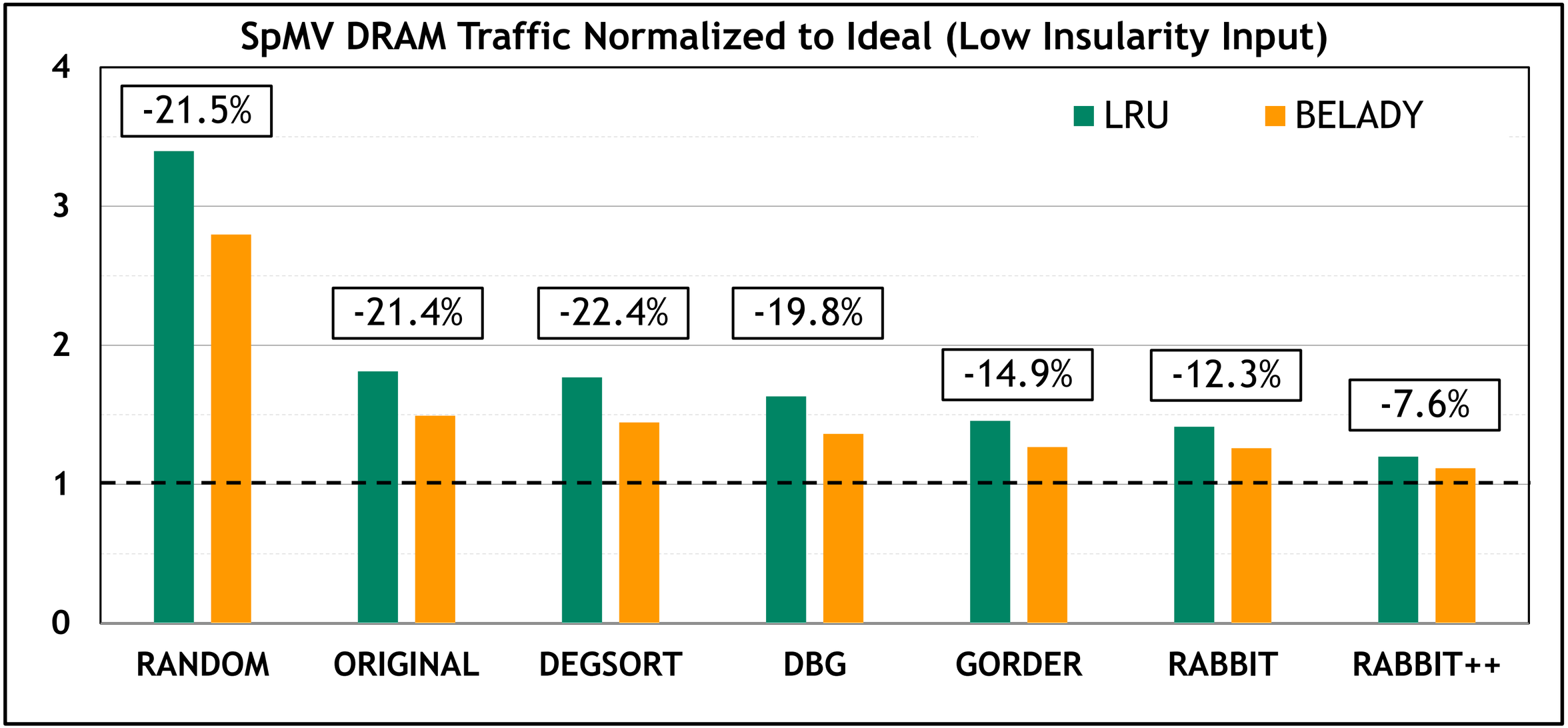


QUESTION: How much additional headroom for improvement do we have with RABBIT++ for low-insularity inputs?

RABBIT++ improves SpMV performance over RABBIT by up to 1.57x

HOW FAR IS RABBIT++ FROM OPTIMAL?

Experiments on a L2 cache simulator



RABBIT++ ACROSS DIFFERENT CUSPARSE KERNELS/FORMATS

TABLE IV: **Run time (normalized to ideal) across different cuSPARSE kernels:** *RABBIT++ consistently improves upon RABBIT's ability to bring kernels closer to peak performance.*

	SpMV-COO			SpMM-CSR-4			SpMM-CSR-256		
	ALL	*I <0.95*	*I >= 0.95*	*ALL*	*I <0.95*	*I >= 0.95*	*ALL*	*I <0.95*	*I >= 0.95*
RANDOM	5.37×	4.94×	5.97×	29.33×	32.17×	26.07×	139.3×	196.6×	75.13×
ORIGINAL	1.84×	2.1×	1.55×	5.97×	8.92×	3.58×	26.81×	43.79×	10.99×
RABBIT	1.49×	1.73×	**1.23×**	4.31×	7.39×	**2.18×**	20.32×	50.3×	**3.91×**
RABBIT++	**1.4×**	**1.55×**	**1.23×**	**3.79×**	**5.85×**	**2.18×**	**18.7×**	**43.97×**	3.95×

PREPROCESSING COSTS

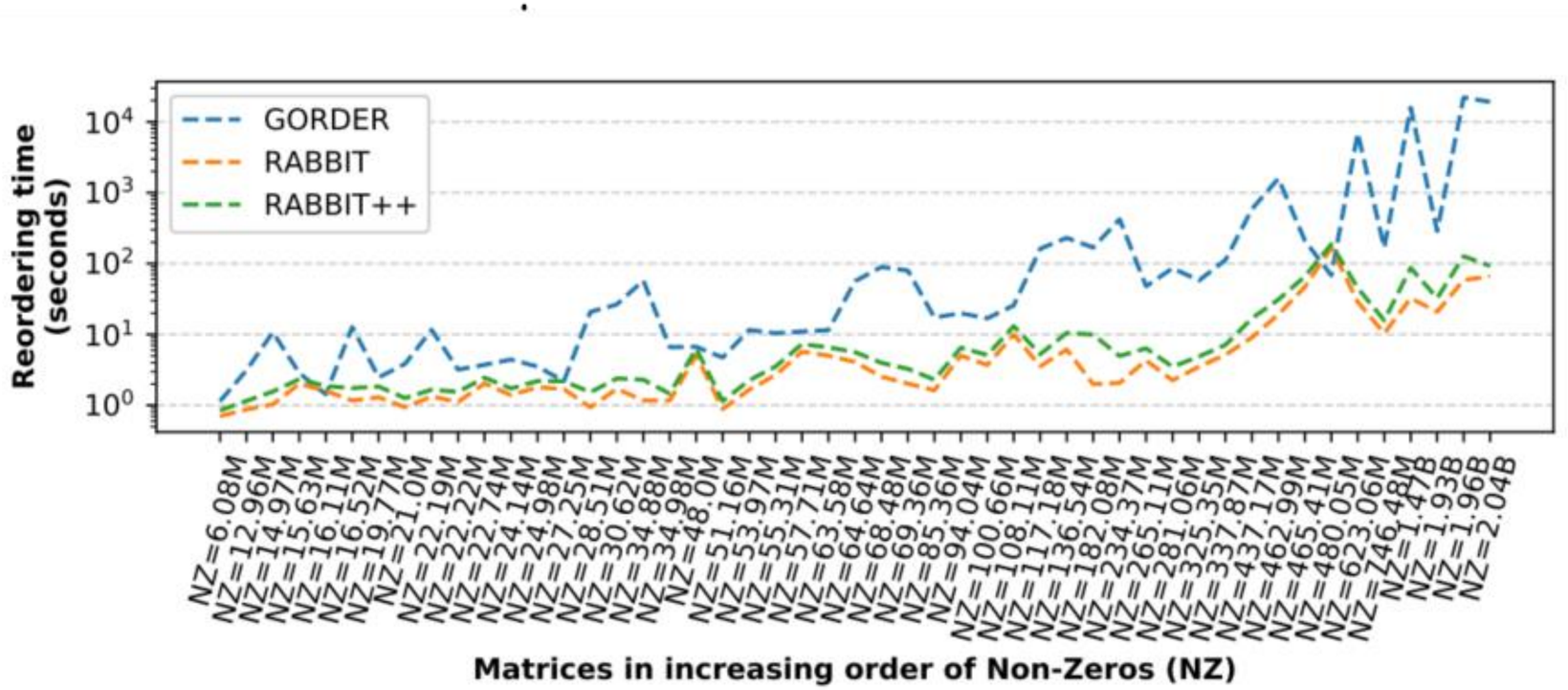


Fig. 9: **Matrix reordering time as the matrix size increases:** *Compared to GORDER, both RABBIT and RABBIT++ are more practical matrix reordering solutions.*

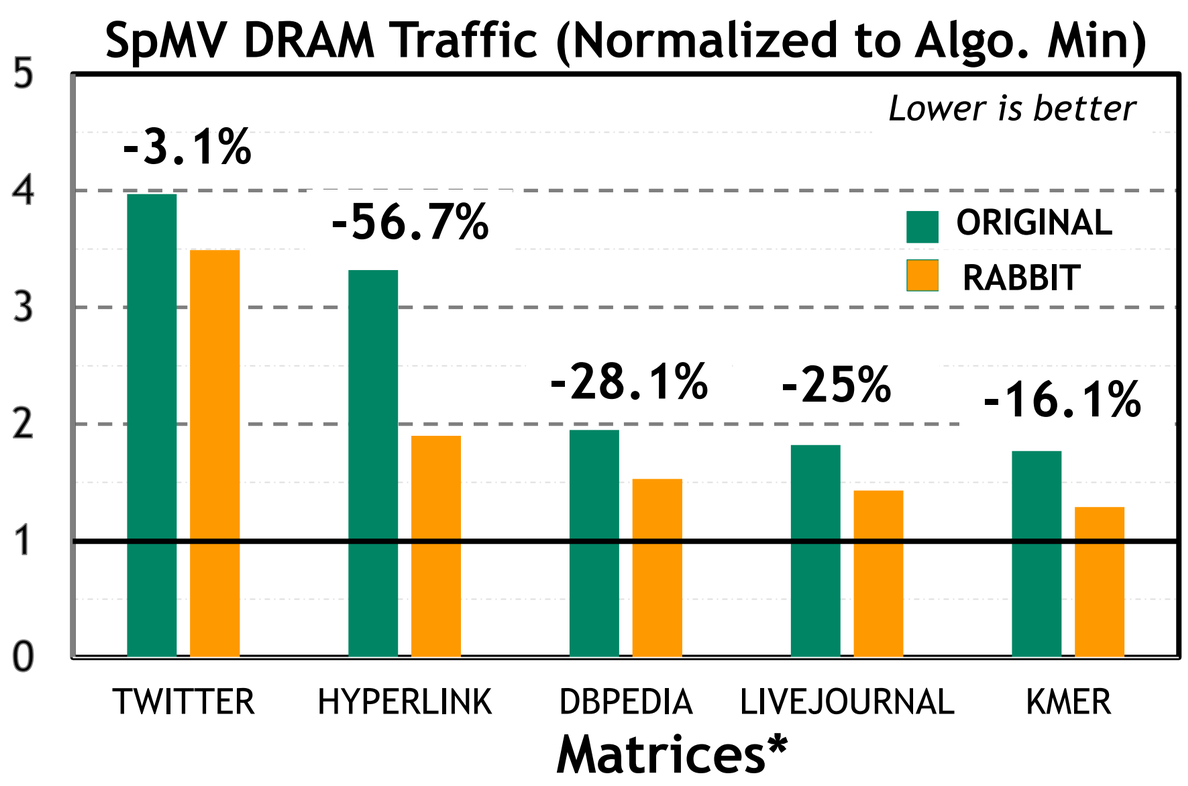
EVALUATION METHODOLOGY

HARDWARE: NVIDIA A6000 GPU

L2 Cache	DRAM Bandwidth	Mem Capacity
6MB	768GB/s	48GB

SOFTWARE: NVIDIA cuSPARSE library (v11.8)

INPUTS:



Locality improvement from reordering is sensitive to matrices

PROBLEM: Prior work on reordering were evaluated on a small number (<10) of arbitrarily-selected matrices

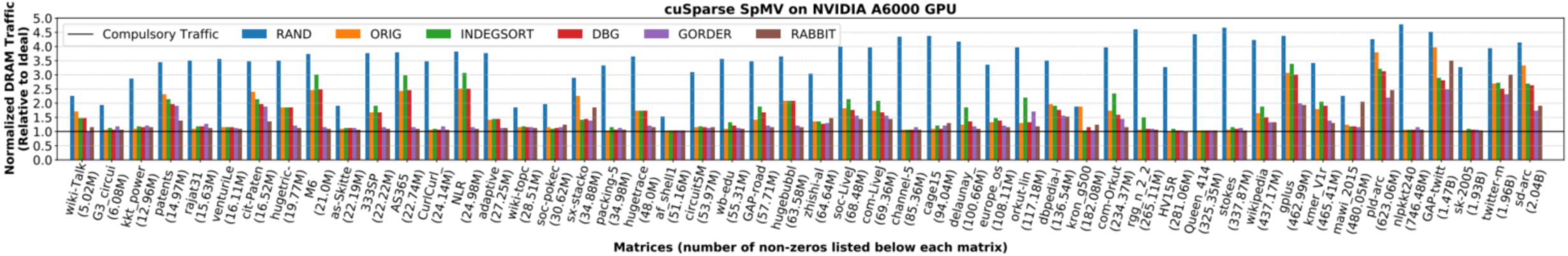
**Subset of matrices evaluated*

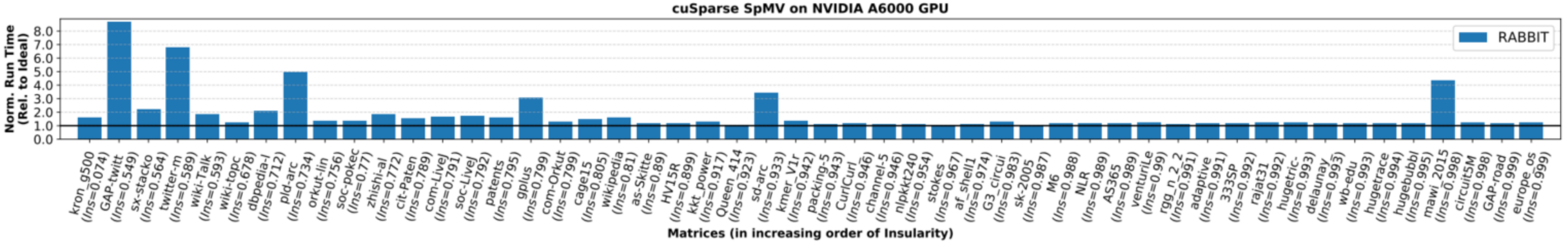
SPMV KERNEL

Algorithm 1 SpMV kernel with sparse matrix in CSR format

```
1: Y ← 0
2: parfor row in [0, |Rows|) do
3:     rowStart ← A.rowOffsets[row]
4:     rowEnd ← A.rowOffsets[row + 1]
5:     for i in [rowStart, rowEnd) do
6:         Y[row] += A.values[i] * X[A.coords[i]]
```

DRAM TRAFFIC AND PERF RESULTS ACROSS ALL INPUTS





DIFFERENT ALTERNATIVES FOR RABBIT++

TABLE II: **Design space of RABBIT modifications:** *SpMV run time (normalized to ideal) when applying different combinations of RABBIT modifications (Figure 5).*

	Without Insular Nodes Grouped			**With Insular Nodes Grouped**		
	ALL-MATS	*INS <0.95*	*INS >= 0.95*	*ALL-MATS*	*INS <0.95*	*INS >= 0.95*
RABBIT	1.54×	1.81×	1.25×	1.49×	1.70×	1.25×
RABBIT+HUBSORT	1.63×	1.89×	1.35×	1.57×	1.86×	1.26×
RABBIT+HUBGROUP	1.48×	1.65×	1.29×	**1.46**×	1.65×	1.25×

RABBIT++ TRAFFIC REDUCTIONS

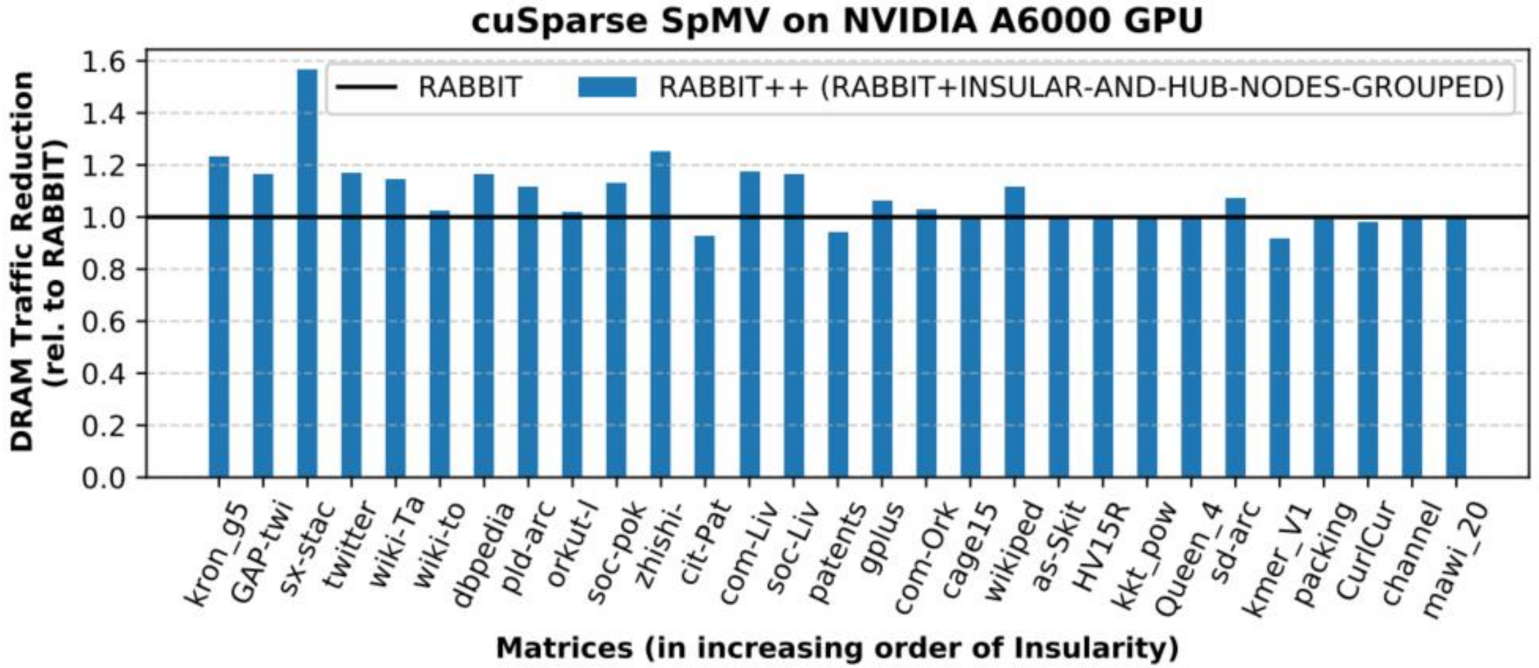


Fig. 7: **Reduction in SpMV's DRAM traffic with RABBIT++:** *In the interest of space, we only include results for matrices with Insularity* < 0.95*). For matrices with Insularity* ≥ 0.95*, RABBIT++'s DRAM traffic is within 1% of RABBIT.*

RABBIT++ ACROSS DIFFERENT CUSPARSE KERNELS/FORMATS

TABLE IV: **Run time (normalized to ideal) across different cuSPARSE kernels:** ***RABBIT++ consistently improves upon RABBIT's ability to bring kernels closer to peak performance.***

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RANDOM	5.37×	4.94×	5.97×	29.33×	32.17×	26.07×	139.3×	196.6×	75.13×
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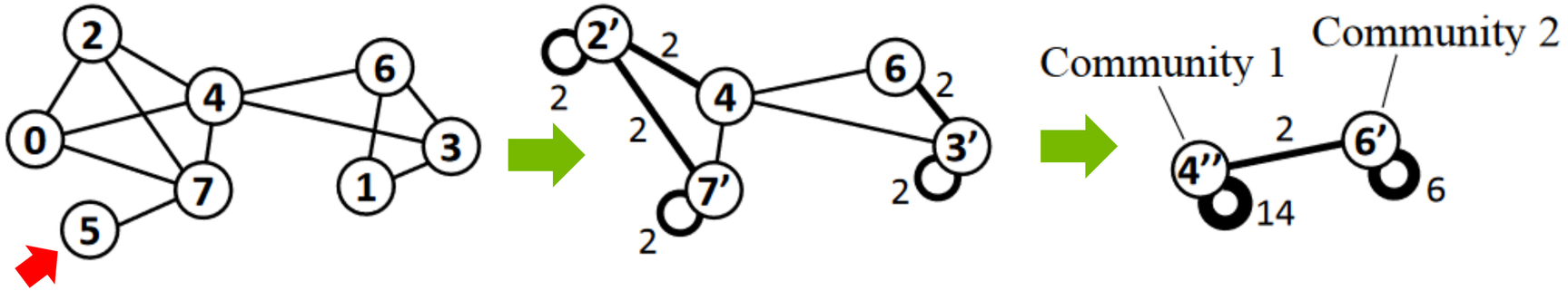
COMMUNITY-BASED GRAPH REORDERING

High-level Overview of RABBIT

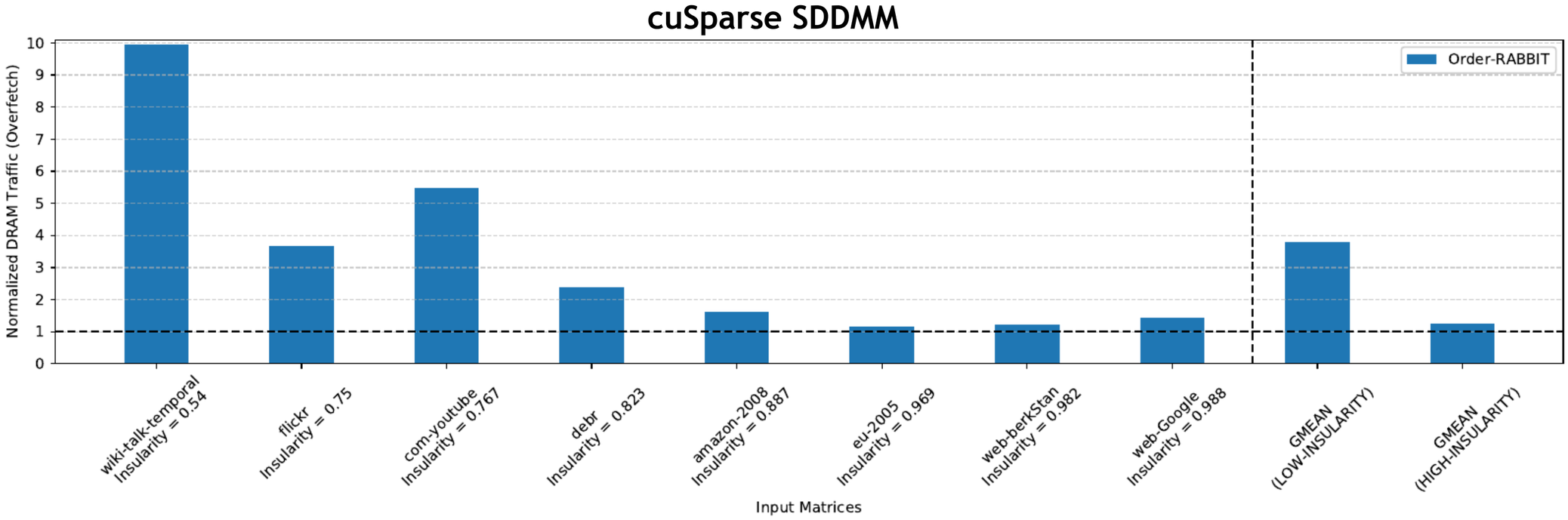


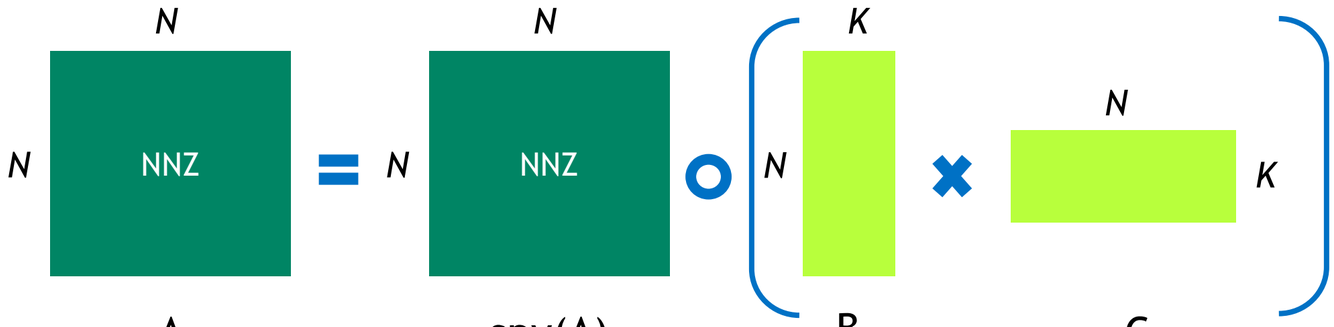
Incremental aggregation on the graph

$$\Delta Q(u,v) = 2\left\{\frac{w_{uv}}{2m} - \frac{d(u)d(v)}{(2m)^2}\right\}$$

Figure from "Rabbit Order: Just-in-time Parallel Reordering for Fast Graph Analysis" IPDPS 2016

RABBIT IMPROVES LOCALITY ACROSS MULTIPLE KERNELS





Insularity

A
(Sparse Matrix)

spy(A)
(Sparse Matrix)

B
(Dense Matrix)

C
(Dense Matrix)

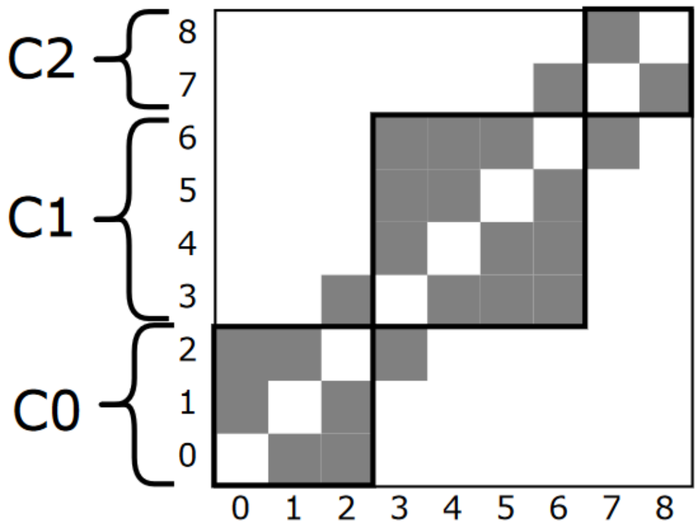
COMPARING EXISTING REORDERING TECHNIQUES

Reordering Technique*	Structural Property Targeted	Intuition
Degree Sorting (DEGSORT)	Power-law degree distribution	Assign IDs in decreasing order of vertex degrees
Degree Based Grouping (DBG)	Power-law degree distribution	Assign contiguous IDs to vertices in the same degrees bucket
RABBIT	Community Structure	Assign contiguous IDs to vertices in the same community
GORDER	Power-Law + Community	Assign contiguous IDs to vertices with a strong overlap in neighborhoods

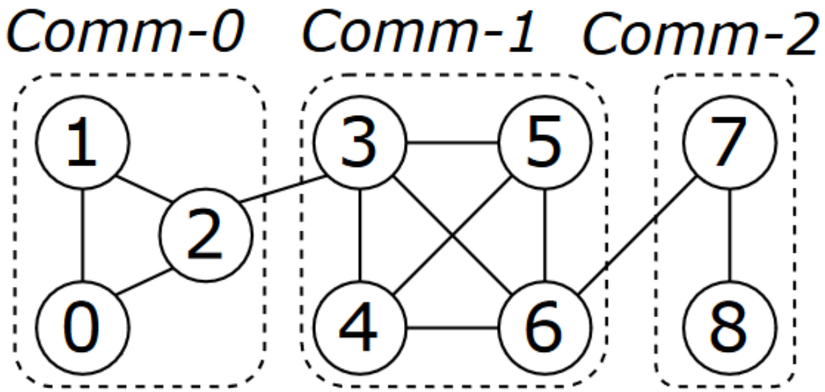
**Techniques organized in increasing order of reordering complexity*

A METRIC TO QUANTIFY BENEFITS WITH RABBIT

Insularity



From a locality perspective, we need **highly self-contained and small** communities

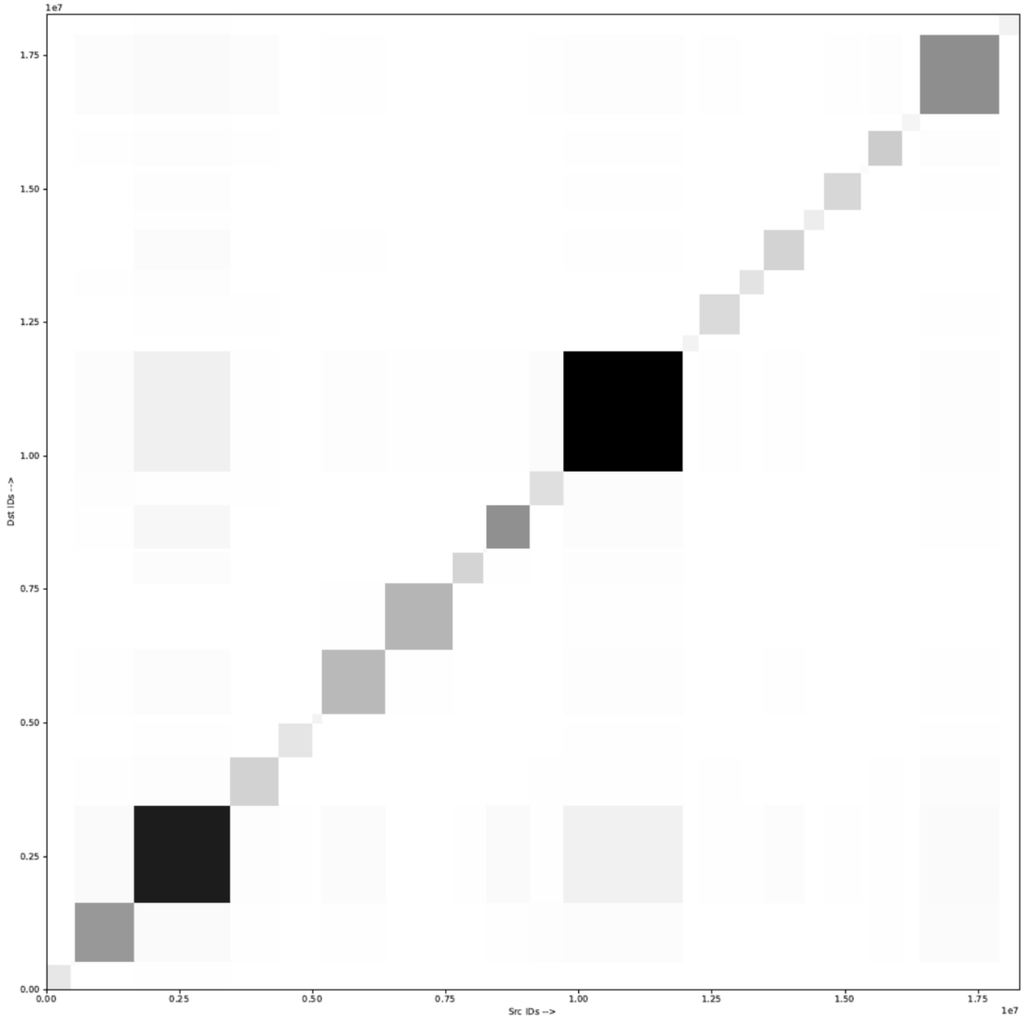


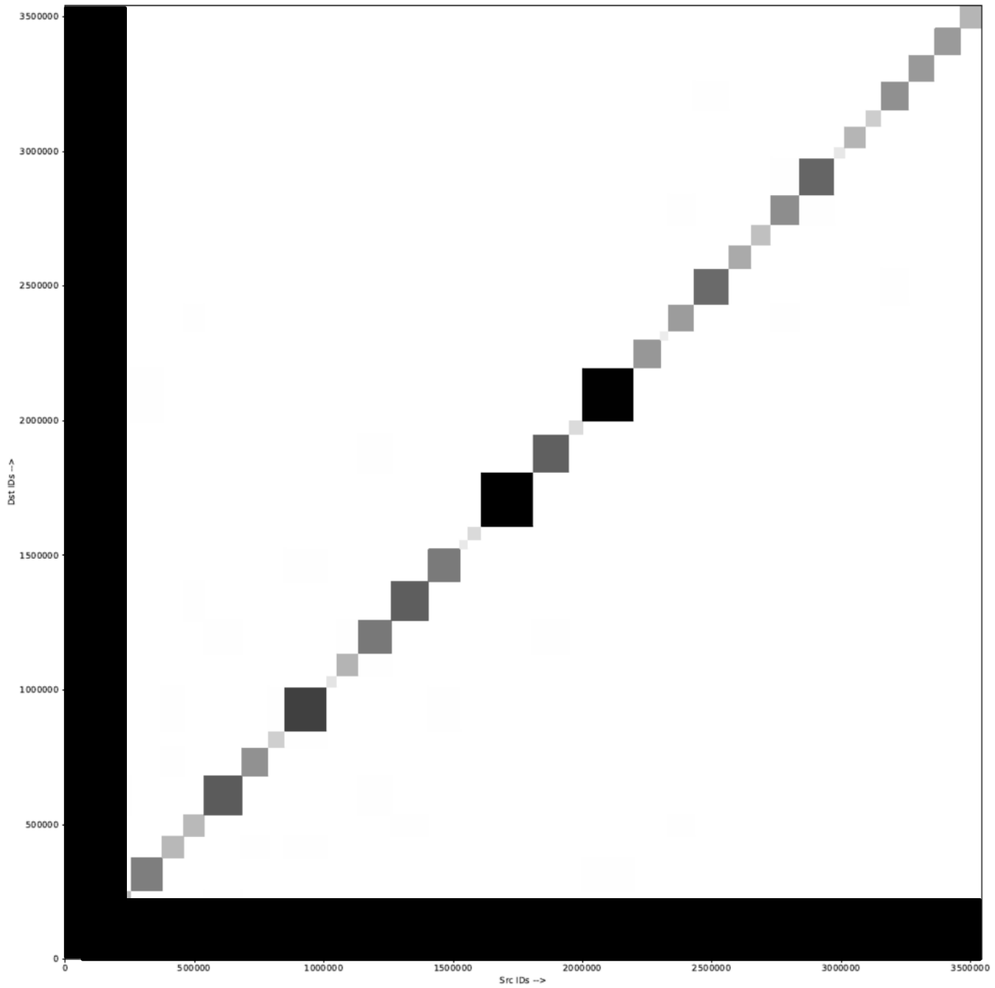
$$\text{Insularity} = \frac{Intra{-}Comm{-}Edges}{Total{-}Comm{-}Edges} = \frac{20}{24} = 0.83$$

SpMV Run Time (normalized to ideal) has a strong inverse correlation with insularity (ρ = -0.74)

ENHANCING RABBIT

**Cartoon representation*





Insular Nodes

Hubs

RABBIT

RABBIT++

MATRIX REORDERING ACROSS DIFFERENT GPUS